M344 15 Nov 18

§16.6 Parametric Surfaces

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$ parametric space curve.

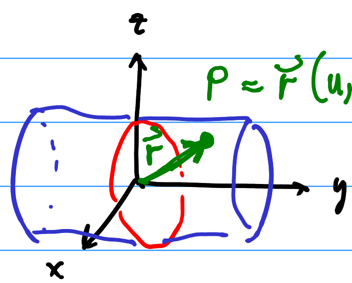
$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle \leftarrow$ parametric surface in $\mathbb{R}^3$.

Ex. $\vec{r} = \langle 2\cos u, v, 2\sin u \rangle$ Describe the surface?

$$\left\{ \begin{array}{l} x = 2\cos u \\ y = v \\ z = 2\sin u \end{array} \right. \rightarrow \left\{ \begin{array}{l} x = 2\cos u \\ z = 2\sin u \end{array} \right.$$

$x^2 + z^2 = 4\cos^2 u + 4\sin^2 u = 4$

circle $r=2$ in xz-plane



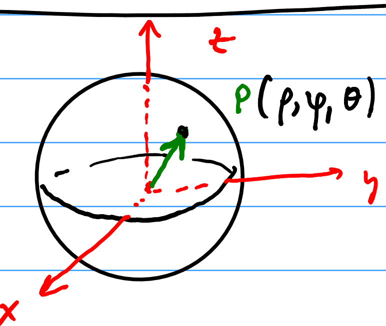
cylinder w/ y-axis as central axis.

Ex. $\vec{r} = \langle (2+\sin v)\cos u, (2+\sin v)\sin u, u + \cos v \rangle$

Q. what does it look like? SageMath.

Ex. $x^2 + y^2 + z^2 = 1 \swarrow \rho$

parametrize it!



Spherical coord's:

$$\left\{ \begin{array}{l} x = \rho \sin\varphi \cos\theta \\ y = \rho \sin\varphi \sin\theta \\ z = \rho \cos\varphi \end{array} \right.$$

so,

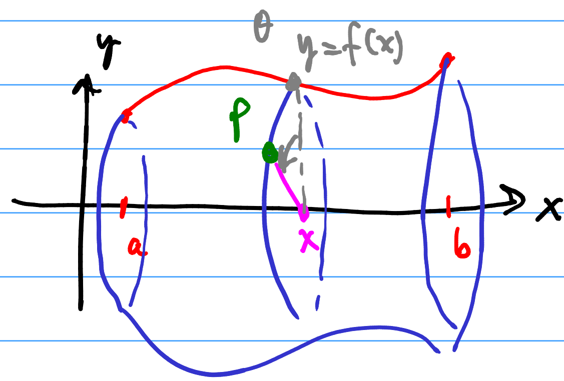
$(*)$ $\left[\begin{array}{l} \vec{r}(\varphi,\theta) = \langle \sin\varphi\cos\theta, \sin\varphi\sin\theta, \cos\varphi \rangle \\ 0 \leq \varphi \leq \pi, \quad 0 \leq \theta < 2\pi \end{array} \right.$

Before: $x^2+y^2+z^2=1 \rightarrow z=\pm\sqrt{1-x^2-y^2}$

$$\begin{cases} f_u(x,y)=\sqrt{1-x^2-y^2} \\ f_\ell(x,y)=-f_u(x,y) \end{cases}$$

Let's plot both and compare!

Ex. Surfaces of revolution

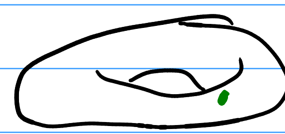


$y=f(x)$

$f(x)>0$ on $[a,b]$

$$\begin{cases} x=x \\ y=f(x)\cdot\cos\theta \\ z=f(x)\cdot\sin\theta \end{cases}$$

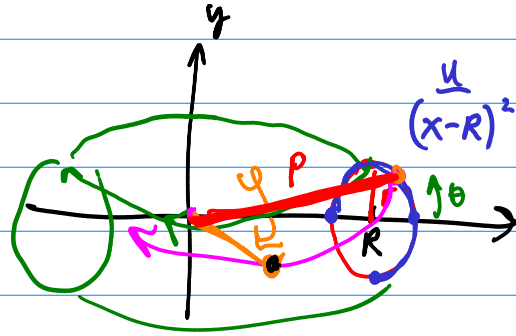
Ex. Parametrize the torus.



$x=\rho\sin\varphi\cos\theta$

$y=\rho\sin\varphi\sin\theta$

$z=\rho\cos\varphi$

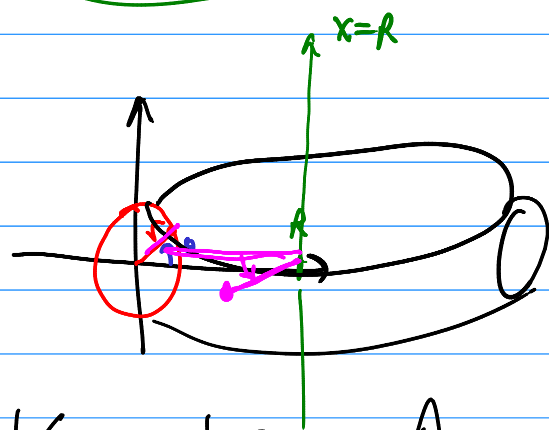


$$(\overset{u}{x-R})^2+\overset{v}{y}{}^2=r^2$$

$x-R=u=r\cos\theta$

$y=v=r\sin\theta$

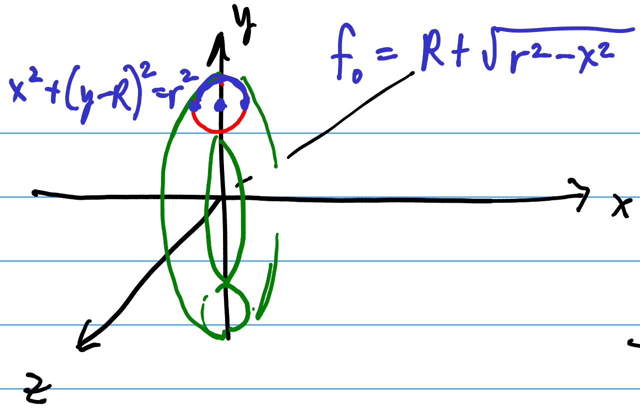
$$\left.\begin{aligned} x&=R+r\cos\theta \\ y&=r\sin\theta \end{aligned}\right]$$



Circle: $x=r\cos\theta$

$y=r\sin\theta$

THIS IS A MESS !!



$$x = x$$
$$y = f(x)\cos\theta$$
$$z = f(x)\sin\theta$$

This is much better :)