Name:______________________________
MA253: Calculus III (Spring 2017)
Instructor: Justin Ryan
Midterm Exam 1: Sections 12.1–12.3, 12*, 13.1–13.4

Read and follow all instructions.

Part I: True or False [1 point each]
Read each statement carefully. In the space provided, write **T** *if the statement is* <u>*always*</u> *true, or* **F** *otherwise.*

______**1.** If the point (a, b) is in the domain of the vector function $z = f(x, y)$, then $\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$.

______**2.** The partial derivative $\partial_x F(a, b)$ represents the slope of the tangent line to the graph of $z = F(x, y)$ at the point $(a, b, F(a, b))$ lying in the plane $y = b$.

______**3.** The unit binormal vector is given by $\mathbf{B}(t) = \mathbf{N}(t) \times \mathbf{T}(t)$.

______**4.** The parabola $y = ax^2$ has curvature $\kappa = 2|a|$ at the origin.

______**5.** The acceleration $\mathbf{a}(t)$ of a particle moving along a space curve always lies in the osculating plane to the space curve.

Part II: Fill in the Blank [1 point each]
Choose the appropriate word or phrase from the word bank, and write its corresponding letter in the space provided.

Word Bank:

A. Acceleration	**B.** Velocity	**C.** Curvature
D. Arc Length	**E.** Torsion	**F.** Ellipses
G. Hyperbolas	**H.** Tangential	**I.** Circles
J. Normal	**K.** Binormal	**L.** Tangent

______**6.** The _______ of a space curve at a point is defined by $\kappa = \left\| \frac{d\mathbf{T}}{ds} \right\|$.

______**7.** Let $v(t)$ denote the speed of a particle moving along a space curve. Then $\dot{v}(t)$ is the _______ component of the particle's acceleration.

______**8.** The level curves of the function $z = 4x^2 + y^2$ are _______ .

______**9.** The _______ of a space curve is defined by $s(t) = \int_0^t \|\dot{\mathbf{r}}(u)\| \, du$.

______**10.** The unit _______ vector to a space curve at a point is given by $\frac{1}{\kappa}\frac{d\mathbf{T}}{ds}$.

Part III: Multiple Choice [5 points each]
Write the letter corresponding to the appropriate answer in the space provided.

11–14. Consider the vector function $\mathbf{r}(t) = \left\langle \frac{1}{1+t^2},\ \sin(t),\ te^t \right\rangle$.

______**11.** What is the domain of $\mathbf{r}$?

A. $(0, \infty)$ **B.** $(-\infty, 0)$

C. $(-\infty, \infty)$ **D.** $[0, \infty)$

______**12.** Compute $\lim_{t \to 0} \mathbf{r}(t)$.

A. $\langle 1, 0, 0 \rangle$ **B.** $\langle 1, 1, 1 \rangle$

C. $\langle 0, 1, 1 \rangle$ **D.** $\langle 1, 1, 0 \rangle$

______**13.** Compute $\dot{\mathbf{r}}(t)$.

A. $\left\langle \frac{-2t}{(1+t^2)^2},\ -\cos t,\ e^t + te^t \right\rangle$ **B.** $\left\langle \frac{1}{2t},\ \cos t,\ e^t \right\rangle$

C. $\left\langle \frac{-2t}{(1+t^2)^2},\ \cos t,\ e^t \right\rangle$ **D.** $\left\langle \frac{-2t}{(1+t^2)^2},\ \cos(t),\ (1+t)e^t \right\rangle$

______**14.** Compute $\int \mathbf{r}(t)\, dt$.

A. $\langle \arctan(t),\ -\cos(t),\ (t-1)e^t \rangle + \mathbf{c}$

B. $\langle \arctan(t),\ \cos(t),\ (t-1)e^t \rangle + \mathbf{c}$

C. $\left\langle \frac{t}{t + \frac{1}{3}t^3},\ -\cos(t),\ \frac{1}{2}t^2 e^t \right\rangle + \mathbf{c}$

D. $\left\langle \frac{t}{t + \frac{1}{3}t^3},\ -\cos(t),\ te^t - e^t \right\rangle + \mathbf{c}$

15–17. Consider the vector function $\mathbf{r}(t) = \langle -t, -\sin t, \cos t \rangle$.

______**15.** Find the arc length function $s(t)$ for $\mathbf{r}$ starting at $t = 0$ and moving in the positive t direction.

A. $s = \sqrt{2}$ **B.** $s = \sqrt{2t}$

C. $s = \sqrt{2}\,t$ **D.** $s = \dfrac{t}{\sqrt{2}}$

______**16.** Reparametrize $\mathbf{r}$ with respect to arc length.

A. $\mathbf{r}(s) = \left\langle -\frac{s}{\sqrt{2}}, -\sin\left(\frac{s}{\sqrt{2}}\right), \cos\left(\frac{s}{\sqrt{2}}\right)\right\rangle$

B. $\mathbf{r}(s) = \langle -s, -\sin(s), \cos(s)\rangle$

C. $\mathbf{r}(s) = \langle -1, -\cos(s), -\sin(s)\rangle$

D. $\mathbf{r}(s) = \left\langle -\frac{1}{\sqrt{2}}, -s\sin\left(\frac{t}{\sqrt{2}}\right), s\cos\left(\frac{t}{\sqrt{2}}\right)\right\rangle$

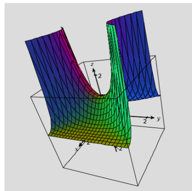
______**17.** Compute the curvature function $\kappa(s)$.

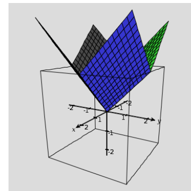
A. $\kappa(s) = \dfrac{s}{\sqrt{2}}$ **B.** $\kappa(s) = s\sqrt{2}$

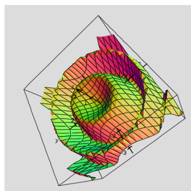
C. $\kappa(s) = \sqrt{2}$ **D.** $\kappa(s) = \dfrac{1}{\sqrt{2}}$

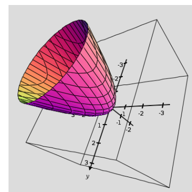
18–20. Choose the graph that best corresponds to the function of two variables.

______**18.** $f(x, y) = e^{-2x^2+y^2}$

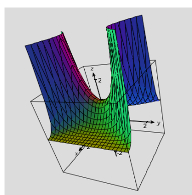
A. 

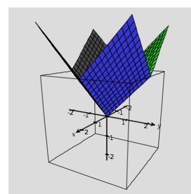
B. 

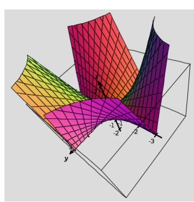
C. 

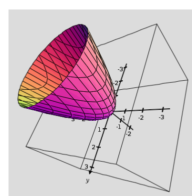
D. 

______**19.** $g(x, y) = |x| + |y|$

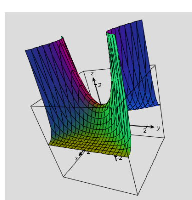
A. 

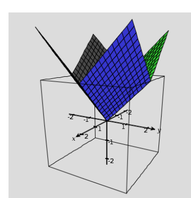
B. 

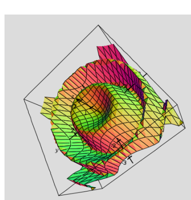
C. 

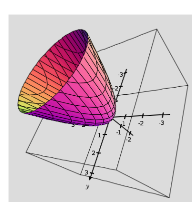
D. 

______**20.** $F(x, y) = \sin(x^2 + y^2)$

A. 

B. 

C. 

D. 

21–22. Consider the function $F(x, y) = e^{xy} \sin y$.

______**21.** Compute $\partial_x F$.

A. $e^{xy} y \sin y$ **B.** $e^{xy} y \cos y$

C. $e^{xy} x \sin y$ **D.** $e^{y} \sin y$

______**22.** Compute $\partial_y F$.

A. $e^{xy} x \cos y$ **B.** $e^{x} \cos y$

C. $e^{xy}(\cos y + x \sin y)$ **D.** $e^{y}(\cos y - x \sin y)$

______**23.** Compute $\dfrac{\partial^2 F}{\partial x \partial y}$ for the function $F(x, y) = x^y$.

A. $\ln(x)x^y$ **B.** yx^{y-1}

C. $\ln(x^y)(y-1)$ **D.** $y \ln(x) x^{y-1}$

______**24.** Consider the relation $x^2 + y^2 + z^2 = xyz$. Find a formula for $\dfrac{\partial z}{\partial x}$.

A. $\dfrac{-2xyz}{-2xyz}$ **B.** $\dfrac{2x + yz}{2z + xy}$

C. $\dfrac{yz - 2x}{xy - 2z}$ **D.** $\dfrac{yz - 2x}{2z - xy}$

Part IV: Short Answer [5 points each]

You must complete four (4) of the problems in this part. Clearly write the word "OMIT" next to the problem that you wish not to be considered. Show enough *work. Clearly mark your final answers. Partial credit given when deserved.*

25. Consider the space curve $\mathbf{r}(t) = \langle \cos t,\, \sin t,\, t \rangle$. Find $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$ at $t = \pi$.

26. Show that the curvature of a circle of radius a is constant, $\kappa = \frac{1}{a}$.

27. Find an equation of the tangent plane to the curve $z = \sqrt{x^2 - y^2}$ at the point $(5, 3, 4)$.

28. Find an equation of the osculating circle to the curve $y = 4x^2 - x^4$ at the origin, $(0, 0)$.

29. Use either the linearization or differentials to approximate the value of $\sqrt{5.01^2 - 2.99^2}$. Leave your answer as a simplified fraction.