Name:______________________________
MA253: Calculus III (Spring 2017)
Instructor: Justin Ryan
Midterm Exam 2: Chapters 13 and 14

Read and follow all instructions.

Part I: Multiple Choice [5 points each]

Choose the appropriate answer to each question.

Answers to this part of the exam only are to be submitted in Canvas by **11:59 pm on Sunday, 16 April 2017**. *The assignment is titled 'Midterm Exam 2: Part I.'*

CAUTION!—The answers in Canvas will not be in the same order that they are listed on this paper.

______**1.** Find the linearization of the function $f(x, y, z) = x^3\sqrt{y^2 + z^2}$ at the point $(2, 3, 4)$.

A. $\dfrac{3x^2(y^2 + z^2) + x^3y + x^3z}{\sqrt{y^2 + z^2}}$

B. $w = 40 + 60(x - 2) + \frac{24}{5}(y - 3) + \frac{32}{5}(z - 4)$

C. $\left\langle 60, \dfrac{24}{5}, \dfrac{32}{5} \right\rangle$

D. $w = 40 + 60x + \frac{24}{5}y + \frac{32}{5}z$

______**2.** Find the gradient of the function $f(x, y, z) = z^2e^{x\sqrt{y}}$.

A. $\langle 0, 0, 1\rangle$

B. $\left(\dfrac{2x^2y + z^2 + 2z\sqrt{y}}{2\sqrt{y}}\right) e^{x\sqrt{y}}$

C. $e^{x\sqrt{y}} \langle z^2, z^2x, z^2y\rangle$

D. $\left\langle z^2\sqrt{y}e^{x\sqrt{y}}, \frac{z^2x}{2\sqrt{y}}e^{x\sqrt{y}}, 2ze^{x\sqrt{y}} \right\rangle$

______**3.** Let $\mathbf{v} = \langle 2, 1, -2\rangle$, $p = (1, 2, 3)$, and $f(x, y, z) = x^2y + x\sqrt{1 + z}$. Calculate $D_{\mathbf{v}}f(p)$.

A. $\dfrac{25}{6}$

B. $\left\langle 6, 1, \dfrac{1}{4} \right\rangle$

C. $\dfrac{25}{2}$

D. $\left\langle \dfrac{2}{3}, \dfrac{1}{3}, -\dfrac{2}{3} \right\rangle$

______**4.** Find the direction of the maximum rate of change of $f(x, y) = x^2y + \sqrt{y}$ at the point $(2, 1)$.

A. $\dfrac{\sqrt{145}}{2}$ **B.** $\left\langle -4, \dfrac{9}{2} \right\rangle$

C. $\left\langle 4, \dfrac{9}{2} \right\rangle$ **D.** $\left\langle 2xy, x^2 + \dfrac{1}{2\sqrt{y}} \right\rangle$

______**5.** Find $\dfrac{\partial z}{\partial x}$ for $\sin(xyz) = x + 2y + 3z$.

A. $\dfrac{1 - yz\cos(xyz)}{xy\cos(xyz) - 3}$ **B.** $\dfrac{1 - yz\sin(xyz)}{xy\sin(xyz) - 3}$

C. $\dfrac{1 + yz\cos(xyz)}{xy\cos(xyz) + 3}$ **D.** $yz\cos(xyz) = 1$

______**6.** Find an equation of the tangent plane to $xy + yz + zx = 3$ at the point $(1, 1, 1)$.

A. $z = 1 + (x - 1) + (y - 1)$ **B.** $z - 1 = -(x + 1) - (y + 1)$

C. $x + y + z = -1$ **D.** $x + y + z = 3$

______**7.** Calculate the iterated integral $\int_1^2 \int_0^2 y + 2xe^y \, dx \, dy$.

A. $3 + 4(e^2 - e)$ **B.** $3 + 4e$

C. $4e(e-1)$ **D.** $e(e-1)$

______**8.** Calculate the iterated integral $\int_0^1 \int_o^y \int_x^1 6xyz \, dz \, dx \, dy$.

A. $\frac{1}{4}$ **B.** $\frac{9}{40}$

C. $\frac{1}{5}$ **D.** $-\frac{1}{4}$

______**9.** Calculate the iterated integral by changing the order of integration. $\int_0^1 \int_x^1 \cos(y^2) \, dy \, dx$.

A. $\frac{1}{2}\sin(1)$ **B.** $\frac{1}{2}\cos(1)$

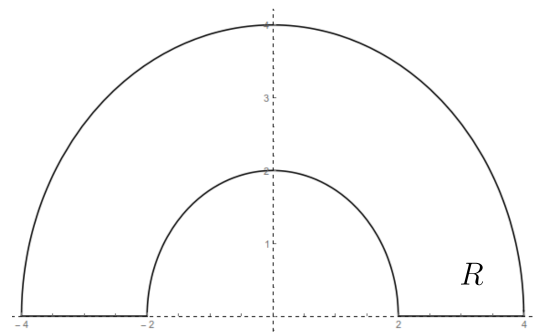
C. 0 **D.** $\frac{1}{2}$

______**10.** Compute $\iint_D \frac{y}{1+x^2}\,dA$ where D is bounded by $y=\sqrt{x}$, $y=0$, and $x=1$.

A. $\frac{1}{4}(\ln 2 - 1)$ **B.** 0

C. $\frac{1}{4}\ln 2$ **D.** $\ln 2^4$

______**11.** Write $\iint_R f(x,y)\,dA$ as an iterated integral, where R is the region shown below and f is continuous on R.



A. $\int_0^{\pi}\int_2^4 f(r,\theta)\,r\,dr\,d\theta$ **B.** $\int_0^{2\pi}\int_0^2 f(r\cos\theta, r\sin\theta)\,dr\,d\theta$

C. $\int_0^{\pi}\int_2^4 f(r\cos\theta, r\sin\theta)\,dr\,d\theta$ **D.** $\int_0^{\pi}\int_2^4 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$

______**12.** Find the volume of the solid bounded above by the surface $z=x^2y$ and sitting above the triangle in the xy-plane with vertices $(1,0)$, $(2,1)$, and $(4,0)$.

A. $\frac{53}{20}$ **B.** $\frac{105}{3}$

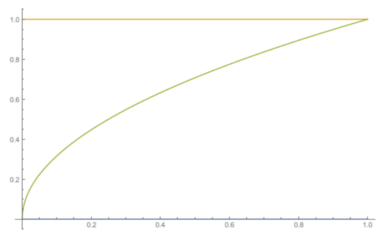
C. $\frac{20}{35}$ **D.** $\frac{53\pi}{20}$

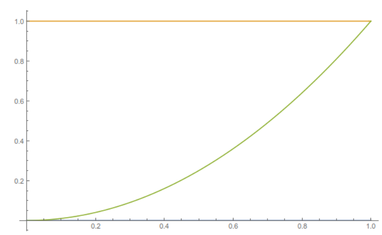
______**13.** Compute the Jacobian of the transformation $T : (u, v, w) \mapsto (uv, vw, uw)$.

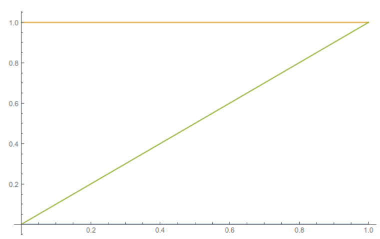
A. 0 **B.** $(uvw)^2$

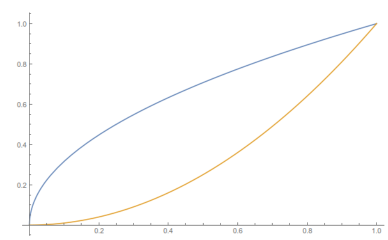
C. $2uvw$ **D.** $\dfrac{uvw}{2}$

______**14.** Let S be the triangle in the uv-plane with vertices $(0, 0)$, $(1, 1)$, and $(0, 1)$, and consider the transformation $T : (u, v) \mapsto (u^2, v)$. Which graph represents the image $T(S)$ in the xy-plane?

A. 

B. 

C. 

D. 

______**15.** The cylindrical coordinates of a point in 3-dimensional space are $(2\sqrt{3}, \frac{\pi}{3}, 2)$. What are the spherical coordinates of this point?

A. $(\sqrt{3}, 3, 2)$ **B.** $(3, \sqrt{3}, 2)$

C. $\left(4, \dfrac{\pi}{6}, \dfrac{\pi}{3}\right)$ **D.** $\left(4, \dfrac{\pi}{3}, \dfrac{\pi}{3}\right)$

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Midterm Exam 2: Chapters 13 and 14, part II

Part II: Short Answer [5 points each]

You must complete all 5 problems in this part. Show enough work and clearly mark your final answers. Partial credit given when deserved.

Solutions to these problems are to be written on this paper and submitted at the **beginning of class (4:00 pm) on Wednesday, 19 April 2017**. *Late solutions will not be accepted.*

16. Use the method of Lagrange multipliers to find the maximum and minimum value of the function $f(x, y) = \frac{1}{x} + \frac{1}{y}$ subject to the constraint $\frac{1}{x^2} + \frac{1}{y^2} = 1$.

17. Suppose f is a differentiable function of two variables at a point p. Prove that $D_{\mathbf{u}}f(p)$ is maximum when $\mathbf{u}$ is in the same direction as the gradient $\nabla f(p)$. What is the maximum value? Sketch a picture.

18. Use the transformation $T : (u, v, w) \mapsto (u^2, v^2, w^2)$ to find the volume of the region bounded by the surface $\sqrt{x} + \sqrt{y} + \sqrt{z} = 1$ and the coordinate planes.

19. Evaluate the integral

$$\iiint_B (x^2 + y^2 + z^2)\, dV$$

where B is the closed unit ball $x^2 + y^2 + z^2 \leq 1$.

20. Evaluate the integral

$$\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz \, dz \, dx \, dy.$$