Name:______________________________

M344: Calculus III (Spring 2018)
Instructor: Justin Ryan
Unit I Exam: Chapters 12 and 13

WICHITA STATE UNIVERSITY

Read and follow all instructions.

Part I: True or False [2 point each]
Read each statement carefully. In the space provided, write **T** *if the statement is* <u>*always*</u> *true, or* **F** *otherwise.*

______**1.** The vector $\mathbf{x} = \langle 1, 2, 3\rangle$ is parallel to the plane $2x + 4y + 6z = -7$.

______**2.** The arc length function of a space curve is defined by $s(t) = \int_0^t \|\dot{\mathbf{r}}(u)\|\, du$.

______**3.** The unit binormal vector is given by $\mathbf{B}(t) = \mathbf{N}(t) \times \mathbf{T}(t)$.

______**4.** The parabola $y = ax^2$ has curvature $\kappa = 2a$ at the origin.

______**5.** The unit unit normal vector to a space curve at a point is given by $\mathbf{N}(s) = \kappa \dfrac{d\mathbf{T}}{ds}$.

Part II: Multiple Choice [5 points each]
Write the letter corresponding to the appropriate answer in the space provided.

6 – 7. Consider the line parametrized by the vector function $\mathbf{r}(t) = \langle -t, 3 - t, 2t + 1\rangle$.

______**6.** What is the direction vector of the line?

A. $\langle 0, 3, 2\rangle$ **B.** $\langle -1, -1, 2\rangle$

C. $\langle -1, 3, 2\rangle$ **D.** $\langle -1, -1, 1\rangle$

______**7.** Find the distance between the line and the point $Q(0, 0, 1)$.

A. 3 **B.** $\dfrac{15}{2}$

C. $\dfrac{\sqrt{30}}{2}$ **D.** $\dfrac{\sqrt{15}}{2}$

8–11. Consider the vector function $\mathbf{r}(t) = \left\langle \frac{1}{1+t^2}, \sin(t), te^t \right\rangle$.

_______**8.** What is the domain of $\mathbf{r}$?

A. $(0, \infty)$ **B.** $(-\infty, 0)$

C. $(-\infty, \infty)$ **D.** $[0, \infty)$

_______**9.** Compute $\lim_{t \to 0} \mathbf{r}(t)$.

A. $\langle 1, 0, 0 \rangle$ **B.** $\langle 1, 1, 1 \rangle$

C. $\langle 0, 1, 1 \rangle$ **D.** $\langle 1, 1, 0 \rangle$

_______**10.** Compute $\dot{\mathbf{r}}(t)$.

A. $\left\langle \frac{-2t}{(1+t^2)^2}, -\cos t, e^t + te^t \right\rangle$ **B.** $\left\langle \frac{1}{2t}, \cos t, e^t \right\rangle$

C. $\left\langle \frac{-2t}{(1+t^2)^2}, \cos t, e^t \right\rangle$ **D.** $\left\langle \frac{-2t}{(1+t^2)^2}, \cos(t), (1+t)e^t \right\rangle$

_______**11.** Compute $\int \mathbf{r}(t)\, dt$.

A. $\langle \arctan(t), -\cos(t), (t-1)e^t \rangle + \mathbf{c}$

B. $\langle \arctan(t), \cos(t), (t-1)e^t \rangle + \mathbf{c}$

C. $\left\langle \frac{t}{t + \frac{1}{3}t^3}, -\cos(t), \frac{1}{2}t^2 e^t \right\rangle + \mathbf{c}$

D. $\left\langle \frac{t}{t + \frac{1}{3}t^3}, -\cos(t), te^t - e^t \right\rangle + \mathbf{c}$

12–14. Consider the vector function $\mathbf{r}(t) = \langle -t, -\sin t, \cos t \rangle$.

_______**12.** Find the arc length function $s(t)$ for $\mathbf{r}$ starting at $t = 0$ and moving in the positive t direction.

A. $s = \sqrt{2}$ **B.** $s = \sqrt{2t}$

C. $s = \sqrt{2}\,t$ **D.** $s = \dfrac{t}{\sqrt{2}}$

_______**13.** Reparametrize $\mathbf{r}$ with respect to arc length.

A. $\mathbf{r}(s) = \left\langle -\dfrac{s}{\sqrt{2}}, -\sin\left(\dfrac{s}{\sqrt{2}}\right), \cos\left(\dfrac{s}{\sqrt{2}}\right) \right\rangle$

B. $\mathbf{r}(s) = \langle -s, -\sin(s), \cos(s) \rangle$

C. $\mathbf{r}(s) = \langle -1, -\cos(s), -\sin(s) \rangle$

D. $\mathbf{r}(s) = \left\langle -\dfrac{1}{\sqrt{2}}, -s\sin\left(\dfrac{t}{\sqrt{2}}\right), s\cos\left(\dfrac{t}{\sqrt{2}}\right) \right\rangle$

_______**14.** Compute the curvature function $\kappa(s)$.

A. $\kappa(s) = \dfrac{s}{\sqrt{2}}$ **B.** $\kappa(s) = s\sqrt{2}$

C. $\kappa(s) = \sqrt{2}$ **D.** $\kappa(s) = \dfrac{1}{\sqrt{2}}$

Part III: Short Answer [10 points each]
You must complete all five problems in this part. Show <u>*enough*</u> *work. Clearly mark your final answers. Partial credit given when deserved.*

15. Consider the space curve $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$. Find **T**, **N**, and **B** at $t = \pi$.

16. Show that the curvature of a circle of radius a is constant, $\kappa = \frac{1}{a}$.

17. Find an equation of the osculating circle to the curve $y = 4x^2 - x^4$ at the origin, $(0,0)$.

18. Prove that the unit tangent vector field $\mathbf{T}$ of a smooth space curve satisfies $\dot{\mathbf{T}}(t) \perp \mathbf{T}(t)$ for all t in the domain of $\mathbf{T}$.

19. Find an equation of the plane passing through the points $P(1,0,0)$, $Q(0,2,0)$, and $R(0,0,3)$.