Name:_____
M344: Calculus III (Spring 2018)
Instructor: Justin Ryan
Unit II Exam: Chapter 14

WICHITA STATE UNIVERSITY

Instructions. *You must answer all of the problems in part I. If you wish, you may omit one of the problems in part II. If all 5 problems in part II are to be graded, then each problem is worth 10 points and the minimum score is 10. If you choose to omit a problem, then each problem is worth 15 points and the minimum score is 0. Please initial one option below.*

_____ *I would like all 5 problems in part II to be graded.*

_____ *I have omitted one of the problems in part II.*

Part I. Multiple Choice [5 points each]
Choose the appropriate answer to each question and write the corresponding letter neatly on the line provided.

_____**1.** What is the domain of the function $f(x,y) = \dfrac{\sin(xy)}{\sqrt{1-x^2-y^2}}$?

A. $x^2+y^2 \leq 1$ **B.** $\mathbb{R}$

C. $x^2+y^2 < 1$ **D.** $x^2+y^2 > 1$

_____**2.** The following limit does not exist. Along which pair of paths do the limit produce *different* finite real number answers?

$$\lim_{(x,y)\to(0,0)} \frac{xy^4}{y^6+x^3}$$

A. $y=0$; $x=0$ **B.** $x=0$; $x=y^2$

C. $x=0$; $x=y$ **D.** I don't know, dude

3–6. Consider the function $f(x,y) = \dfrac{\sin(x-y)}{1+y^2}$ at the point $P(0,0)$.

_____**3.** Find the differential df.

A. $\dfrac{-2y\sin(x-y)}{(1+y^2)^2}\,dz$

B. $\left\langle \dfrac{\cos(x-y)}{y^2+1}, \dfrac{-(1+y^2)\cos(x-y)-2y\sin(x-y)}{(1+y^2)^2} \right\rangle$

C. $\dfrac{\cos(x-y)}{y^2+1}\,dx + \left(\dfrac{-2y\sin(x-y)}{2y}\right)dy$

D. $\dfrac{\cos(x-y)}{y^2+1}\,dx + \left(\dfrac{-(1+y^2)\cos(x-y)-2y\sin(x-y)}{(1+y^2)^2}\right)dy$

_____**4.** Find the gradient $\nabla f(P)$.

A. $\langle 1,-1\rangle$ **B.** $\langle -1,1\rangle$

C. $\langle 1,1\rangle$ **D.** $\langle 0,1\rangle$

_____**5.** Find the directional derivative of f in the direction $\mathbf{v} = \langle 1,-1\rangle$ at the point $P(0,0)$.

A. 2 **B.** 0

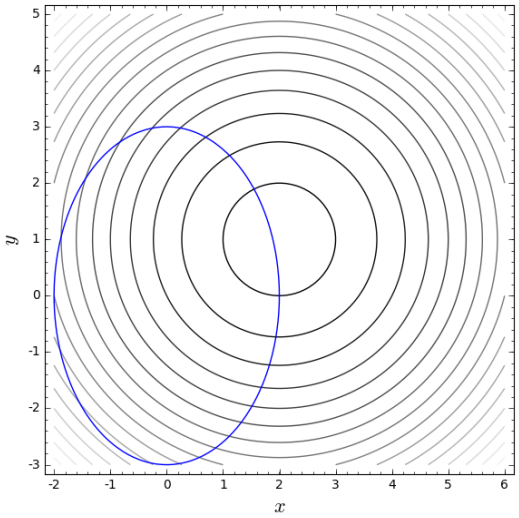
C. $\sqrt{2}$ **D.** $-\sqrt{2}$

_____**6.** Find $\dfrac{\partial^2 f}{\partial x^2}$.

A. $\dfrac{-\cos(x-y)}{1+y^2}$ **B.** $\dfrac{-\sin(x-y)}{1+y^2}$

C. $-\dfrac{2y\cos(x-y)}{(y^2+1)^2} + \dfrac{\sin(x-y)}{y^2+1}$ **D.** $\dfrac{-\sin(x-y)}{2y}$

_____**7.** The image below shows 20 level curves for a function f together with a single level curve $g(x,y) = k$ representing a constraint. At which point does f appear to have a local maximum or minimum under the constraint $g(x,y) = k$?



A. $(0,0)$ **B.** $(2,1)$

C. $(-1.2,-2.5)$ **D.** $(0,-3)$

Part II. Written Problems

You may omit one of the following questions if you'd like, although you are not required to do so. (See the instructions on page 1.) If you do choose to make an omission, clearly indicate which problem you would like to be omitted. Show enough work.

9. Prove the theorem:

Let f be a smooth function of at least two variables and P be a fixed point in the domain of f. Then the directional derivative $D_{\mathbf{u}}f(P)$ is maximum when $\mathbf{u}$ is in the same direction as $\nabla f(P)$ and its maximum value is $\|\nabla f(P)\|$.

10–11. Consider the function $f(x, y) = x^2 - 4x + y^2 + 2y$.

10. Find all critical points of f in the region $x^2 + y^2 < 9$ and determine whether they correspond to local minima, local maxima, or saddle points.

11. Find the absolute maximum and minimum values of f subject to the constraint $x^2 + y^2 = 9$.

12–13. Consider the function $g(x, y) = \ln\left(x^2 + \sin(xy)\right) - x^y$.

12. At the point $(x_0, y_0) = (1, 0)$, what is the maximum value of the directional derivative $D_{\mathbf{u}}g$?

13. Compute the second derivative $\dfrac{\partial^2 g}{\partial x \partial y}$.