Name:_____
M344: Calculus III (Spring 2018)
Instructor: Justin Ryan
Unit III Exam (Take Home): Chapter 15

WICHITA STATE UNIVERSITY

Read and follow all instructions. You may use any resources you want, but make sure you write your work in your own style, show enough work, and provide sufficient explanation when appropriate. These questions are worth 8 points each.

1. Find the average value of the function $f(x) = \int_x^1 \cos(t^2)\,dt$ on the interval $[0,1]$.

Recall that the average value of a continuous function f on an interval $[a,b]$ is given by $f_{\text{avg}} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$.

2. Show that if f is continuous then

$$\int_0^x \int_0^y \int_0^z f(t)\,dt\,dz\,dy = \frac{1}{2}\int_0^x (x-t)^2 f(t)\,dt.$$

Use this fact to evaluate the triple integral

$$\int_0^1 \int_0^y \int_0^z e^{(1-t)^3}\,dt\,dz\,dy.$$

3. Find the volume of the solid region bounded above by the cone $z = 2 - \sqrt{x^2 + y^2}$ and below by the cone $z = \sqrt{x^2 + y^2}$.

4. Evaluate the integral by making an appropriate change of variables.

$$\iint_R \frac{x-2y}{3x-y}\,dA,$$

where R is the parallelogram enclosed by the lines $x-2y=0$, $x-2y=4$, $3x-y=1$, and $3x-y=8$.

5. The surfaces $\rho(\varphi,\theta) = 1 + \frac{1}{5}\sin(m\theta)\sin(n\varphi)$ have been used as models for tumors.

a.) Use a computer to graph the "bumpy sphere" with $m = 6$ and $n = 5$. Print the graph and attach it to your submission.

b.) Write down a triple integral that represents the volume of the bumpy sphere with $m = 6$ and $n = 5$.

c.) Use a computer to evaluate the integral you obtained in part *b*.