Name:______________________________
M344: Calculus III (Spring 2018)
Instructor: Justin Ryan
Final Exam Review

Read and follow all instructions. You may use any resources that you'd like, but you must show sufficient work to receive credit. If turned in, this assignment will be extra credit added to your exam grade. This review is due at the beginning of the final exam.

You should also study the unit in-class exams to prepare for the final exam.

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1. Prove the theorem: *If* $\|\mathbf{r}(t)\| = C$ *for all* t, *where* C *is a constant, then* $\dot{\mathbf{r}}(t) \perp \mathbf{r}(t)$ *for all* t.

2. Compute the arc length function $s(t)$ for the curve $\mathbf{r}(t) = \langle \sin t, -2t, -\cos t \rangle$ starting at $t = 1$ and moving in the positive t direction.

3. Find the equation of the osculating circle to the parabola $y = x^2 - 4x + 5$ at the vertex.

4. Find the unit tangent, normal, and binormal vectors to the curve $\mathbf{r}(t) = \langle t, \sin t, -\cos t \rangle$ at $t = -\frac{\pi}{2}$.

5. Prove that the curvature of a circle of radius $a > 0$ is constant, $\kappa(t) = 1/a$.

6. Give an equation of the normal plane to the curve $\mathbf{r}(t) = \left\langle t, t^2, -t^3 \right\rangle$ at the point $(-2, 4, 8)$.

7. Let f be a smooth function. Prove that the directional derivative $D_{\mathbf{u}} f(p)$ is maximized when $\mathbf{u}$ is in the same direction as the gradient vector $\nabla f(p)$. What is its maximum value?

8. Compute the gradient of the function $f(x, y, z) = e^{x \sin y} \arctan(z)$.

9. Find the maximum and/or minimum values of the function $f(x, y) = x^2 - 4x + y^2 + 6x$ on the open disk $x^2 + y^2 < 16$.

10. Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x^2 - 4x + y^2 + 6x$ along the boundary curve $x^2 + y^2 = 16$.

11. Let $f(x,y)$ be a smooth function, and suppose $x = r\cos\theta,\ y = r\sin\theta$. Use the chain rule to write out a formula for $\dfrac{\partial^2 f}{\partial r \partial\theta}$.

12. Evaluate the integral $\displaystyle\iint_R x^2 \sin(xy)\, dA$ where $R = [0, \frac{\pi}{2}] \times [0,1]$.

13. Use the transformation $u = x - y$, $v = x + y$ to evaluate the integral

$$\iint_R \frac{x-y}{x+y}\, dA$$

where R is the square with vertices $(0,2)$, $(1,1)$, $(2,2)$, and $(1,3)$.

14. Evaluate the double integral $\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx$.

15. Find the volume of the solid bounded above by the cone $z = \sqrt{x^2 + y^2}$ and below by the paraboloid $z = x^2 + y^2$.

16. Find the volume of the solid bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 4$.

17. Evaluate the path integral using your favorite method, where C is the positively-oriented unit circle.

$$\int_C \left(2y - e^{\sin(x^{35})}\right)\, dx + \left(\arctan(y^2 - \ln(y)) + 4x\right)\, dy$$

18. Use Green's Theorem to compute the area of an ellipse with major radius a and minor radius b.

19. Find div **F** and curl **F** for $\mathbf{F}(x, y, z) = e^{yz}\,\mathbf{i} - e^{xz}\,\mathbf{j} + e^{xy}\,\mathbf{k}$.

20. Give an example of Calculus III that you'd encountered in your other class, and/or other reading/studies. Do you anticipate using Calc III in your future work?