**Name**:________________________________

**M555: Differential Equations I** (Spring 2018)
Instructor: Justin Ryan
Unit I Exam: Chapters 1 and 2

WICHITA STATE UNIVERSITY

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**Instructions.** *You must complete problems 1, 2, and 3. If you wish, you may omit one of problems 4, 5, or 6. If all 6 problems are to be graded, then each problem is worth 15 points and the minimum score is 10. If you choose to omit a problem, then each problem is worth 20 points and the minimum score is 0. Please initial one option below.*

_______ *I would like all 6 problems to be graded.*

_______ *I have omitted one of problems 4, 5, or 6.*

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**Part I.** *Complete all 3 problems, showing enough work.*

**1.** Find the general solution of the differential equation.

$$(2y + x^2 y)\, y' = x$$

**2.** Consider the differential equation,

$$\left(2y+1\right)+\left(x-\frac{y}{x}\right)y'=0.$$

*a.*) Show that the DE is not exact as written.

*b.*) Show that the DE is exact when multiplied by the integrating factor $\mu(x,y)=x$, and use this fact to find the general solution of the DE.

**3.** Consider the initial value problem (IVP)

$$\begin{cases} y' = 2xy^2, \\ y(0) = -1. \end{cases}$$

*a.*) Use the Fundamental Existence and Uniqueness Theorem to verify that a solution exists for the given initial data.

*b.*) Solve the IVP and clearly state the domain of the solution.

**Part II.** *You may omit one of the following questions if you'd like, although you are not required to do so. (See the instructions on page 1.) If you do choose to make an omission, clearly indicate which problem you would like to be omitted.*

**4.** Solve the initial value problem.

$$\begin{cases} y' + \dfrac{1}{t}\, y = e^{2t}, \\ y\left(\frac{1}{2}\right) = 4. \end{cases}$$

**5.** Find the particular solution of the initial value problem.

$$\begin{cases} (2xy - y\cos(xy) + \ln y)\,dx + \left(x^2 - x\cos(xy) + \dfrac{x}{y}\right)dy = 0, \\ y(0) = 1 \end{cases}$$

**6.** Consider the initial value problem

$$\begin{cases} y' - y = 1 - t, \\ y(0) = 0. \end{cases}$$

*a.*) Use Picard's Method of Successive Approximations with $\varphi_0 = 0$ to find $\varphi_1$, $\varphi_2$, $\varphi_3$, and a general formula for $\varphi_n$.

*b.*) Find the solution, $\varphi = \lim_{n\to\infty} \varphi_n$.