Name: Key

M555: Differential Equations I (Spring 2018)
Instructor: Justin Ryan
Good Problems 1 – Chapter 1

Instructions *Complete all problems, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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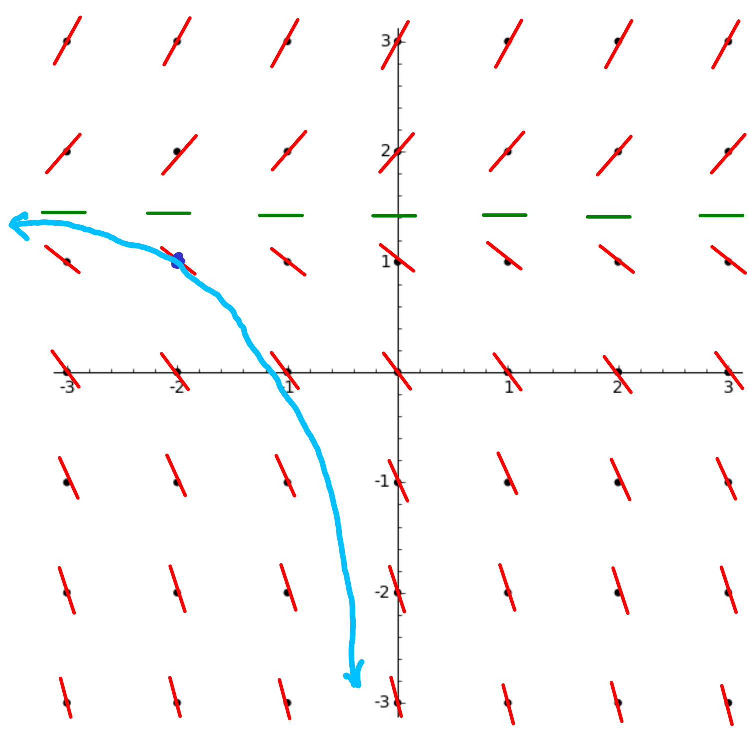
1. Sketch the direction field for the differential equation

$$y' = 2y - 3$$

on the dot paper provided.



y	$y' = 2y-3$
3	3
2	1
1	-1
0	-3
-1	-5
-2	-7
-3	-9

Sketch the integral curve of the DE with initial condition $y(-2) = 1$.

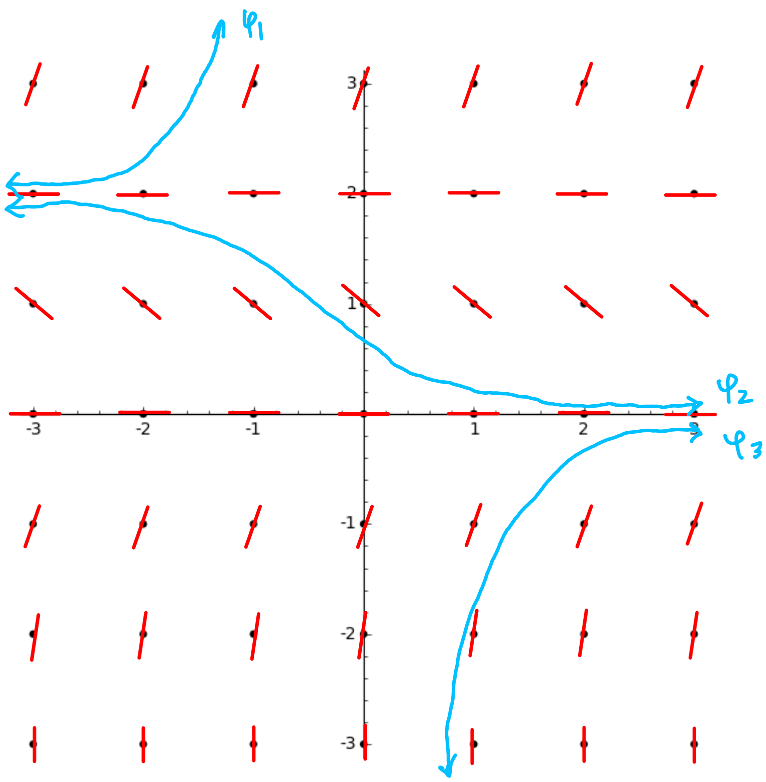
What is the equilibrium solution of this DE? $2y-3=0 \Rightarrow y=\frac{3}{2}$

$\Rightarrow \boxed{y_e=\frac{3}{2}}$

2. Sketch the direction field for the differential equation

$$y' = y(y-2)$$

on the dot paper provided.



y	$y' = y(y-2)$
3	3
2	0
1	-1
0	0
-1	3
-2	8
-3	15

What are the equilibrium solutions? $\varphi = 2$ and $\varphi = 0$.

Sketch three integral curves: one that is bounded only below by an equilibrium solution; one that is bounded only above; and one that is bounded between the equilibria. Give criteria on the initial conditions that will result in each of the cases described above. Can anything else happen?

If $y(t_0) = y_0$ is
$$\begin{cases} >2, & \text{then } \varphi \text{ is bounded below} \\ 0 < y_0 < 2, & \text{then } \varphi \text{ is bounded above and below} \\ <0, & \text{then } \varphi \text{ is bounded above.} \end{cases}$$

No, there are no other possibilities for solutions of this DE.

3. Consider the differential equation $y' = ay - b$, where a and b are fixed real numbers and $a \neq 0$.

a. Find the equilibrium solution φ_e.

b. For any other solution φ of the differential equation, put $Y(t) = \varphi(t) - \varphi_e(t)$. Thus Y measures the deviation of φ from equilibrium. Write down the differential equation satisfied by Y; solve it using principals from Calc I/II; then rearrange to determine φ.

c. Use your result from part *b*) to find a formula for the particular solution to the initial value problem satisfying $y(t_0) = y_0$, where y_0 is not the equilibrium value.

a.) $ay - b = 0 \Rightarrow \boxed{\varphi_e = \frac{b}{a}}$

b.) $Y(t) = \varphi(t) - \frac{b}{a}$, $Y'(t) = \varphi'(t) = a\varphi - b = a\left(\varphi - \frac{b}{a}\right) = aY$.

Thus, Y satisfies the DE $\boxed{Y' = aY}$.

Now, $\frac{dY}{dt} = aY \Rightarrow \frac{1}{Y}dY = a\,dt \Rightarrow \ln Y = at + C \Rightarrow \boxed{Y = Ce^{at}}$

Therefore, $Y = \varphi - \frac{b}{a} = Ce^{at} \Rightarrow \boxed{\varphi = Ce^{at} + \frac{b}{a}}$

c.) $\varphi(t_0) = Ce^{at_0} + \frac{b}{a} = y_0 \Rightarrow C = e^{-at_0}\left(y_0 - \frac{b}{a}\right)$

Therefore, $\varphi(t) = e^{-at_0}\left(y_0 - \frac{b}{a}\right)e^{at} + \frac{b}{a}$

$\Rightarrow \boxed{\varphi(t) = \left(y_0 - \frac{b}{a}\right)e^{a(t-t_0)} + \frac{b}{a}}$