Name:________________________________

M555: Differential Equations I (Spring 2018)

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Good Problems 3 – Sections 2.4, 2.8, and 2.6

WICHITA STATE UNIVERSITY

Instructions *Complete all problems, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. Consider the differential equation

$$y' = \frac{t-y}{2t+5y}.$$

Determine where in the ty-plane the hypotheses of the Fundamental Existence and Uniqueness Theorem are satisfied.

2. Consider the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = t^2 + y^2, \\ y(1) = 2. \end{cases}$$

Make a change of variables to transform this IVP into an equivalent problem with the initial data $y(0) = 0$.

3. Consider the initial value problem

$$\begin{cases} y' = t^2 y - t, \\ y(0) = 0. \end{cases}$$

Use Picard's method of successive approximations with $\varphi_0(t) = 0$ to:

a.) Determine formulas for φ_1, φ_2, φ_3, and φ_4.

b.) Use a computer to plot $\varphi_0, \ldots, \varphi_4$ on the same set of axes. (Include a printed graph with your homework submission.)

c.) Show that the sequence $\{\varphi_n\}$ converges.

4. Consider the initial value problem,

$$\begin{cases} y' - y = 1 - t, \\ y(0) = 0. \end{cases}$$

a.) Use Picard's method of successive iterations to find a formula for the n^{th} approximation, $\varphi_n(t)$.

b.) Compute $\lim_{n\to\infty} \varphi_n(t)$ to find a formula for the actual solution $\varphi(t)$.

c.) Solve the IVP using the method of integrating factors and verify that your solutions using each method agree.

5. Verify that the differential equation is not exact as written, but is exact when multiplied by the given integrating factor, $\mu(x, y)$. Then use the integrating factor to solve the DE.

$$\begin{cases} \left(\dfrac{\sin y}{y} - 2e^{-x}\sin x\right) + \left(\dfrac{\cos y + 2e^{-x}\cos x}{y}\right)\dfrac{dy}{dx} = 0, \\ \mu(x, y) = ye^{x} \end{cases}$$