Name:________________________________

M555: Differential Equations I (Spring 2018)
Instructor: Justin Ryan
Good Problems: Chapter 5

WICHITA STATE UNIVERSITY

Instructions *Complete all problems, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. Determine the a_n so that the equation

$$\sum_{n=1}^{\infty} n a_n x^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$$

is satisfied. Identify the function represented by the power series $\sum_{n=0}^{\infty} a_n x^n$.

2. Find the Taylor series representation for the function

$$f(x) = \frac{1}{1-x}$$

about the point $x_0 = 2$. What is the radius of convergence?

3. Find the Taylor series representation for the function

$$f(x) = \ln(x)$$

about the point $x_0 = 1$. What is the radius of convergence?

5. Find a power series solution of the SODE

$$xy'' + y' + xy = 0$$

about the point $x_0 = 1$. Find at least 4 terms in each solution, and if possible, find the general term for each solution.

6. Consider the IVP

$$\begin{cases} y'' + x^2 y' + (\sin x)\, y = 0, \\ y(0) = a_0, \\ y'(0) = a_1. \end{cases}$$

Assuming $y = \varphi(x)$ is a solution, find expressions for φ'', φ''', $\varphi^{(4)}$, and $\varphi^{(5)}$, and use them to solve for the terms $a_2, \ldots, a_5$ of the power series expansion of φ.

6. Determine a lower bound for the radius of convergence of series solutions to the SODE about each of the points x_0^i.

$$(1+x^3)y'' + 4xy' + y = 0; \quad x_0^1 = 0, \ \ x_0^2 = 1, \ \ x_0^3 = 2$$

7. Identify all singular points of the SODE and determine whether they are regular or irregular.

$$x(1-x^2)^2y'' + (1-x^2)^2y' + 2(1+x)y = 0$$