Name: Key

M555: Differential Equations I (Su.19)
Good Problems 1
Sections 1.1-3, 2.1-2

WICHITA STATE UNIVERSITY

Instructions. Complete all problems, showing enough work. All work must be done on this paper. You may not use any notes or electronic devices. All you need is a pencil and your brain.

(20 pts each) **1.** A field mouse population satisfies the differential equation

$$\frac{dp}{dt} = \frac{p}{2} - 450,$$

where p represents the population (in thousands) at time t (in years). If the initial population of field mice is $p(0) = 450$, find the time that the population will go extinct. Leave your answer in terms of a logarithm.

$$\frac{dp}{dt} = \frac{1}{2}(p-900)$$

$$\int \frac{1}{p-900}\,dp = \int \frac{1}{2}\,dt$$

$$\ln(p-900) = \frac{1}{2}t + C$$

$$p = 900 + Ce^{\frac{1}{2}t}$$

$$p(0) = 900 + C = 450$$

$$C = 450.$$

$$p(t) = 900 - 450e^{\frac{1}{2}t}$$

15 pts

$$P(T) = 0, \quad T = ?$$

$$900 = 450e^{\frac{1}{2}T}$$

$$2 = e^{\frac{1}{2}T}$$

$$\ln 2 = \frac{1}{2}T$$

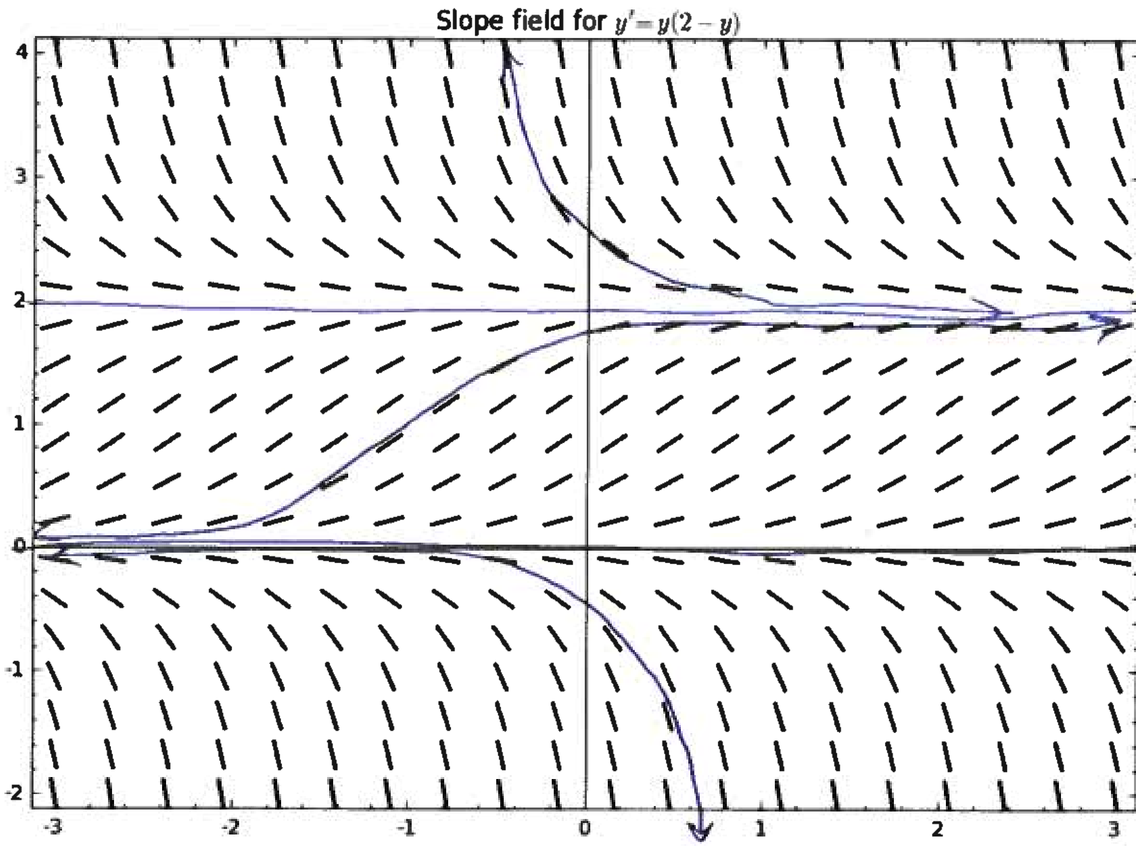
$$2\ln 2 = T \quad \leftarrow \text{ok.}$$

$$\boxed{T = \ln 4}$$

5 pts

2. Given below is the slope field for the differential equation $y' = y(2-y)$. Plot and label the equilibrium solution(s), and sketch three non-equilibrium solution curves with distinctly different behavior.

[10 pts]



[10 pts] Find the general solution of the DE. You do not need to solve for y explicitly.

$y' = y(2-y)$

$\frac{1}{y(2-y)}\,dy = 1\,dt$

PFD: $\frac{1}{y(2-y)} = \frac{A}{y} + \frac{B}{2-y}$

$1 = A(2-y) + By$

$y=0$: $1 = 2A$ $\quad A = 1/2$

$y=2$: $1 = 2B$ $\quad B = 1/2$

$\frac{1}{2}\left(\frac{1}{y} + \frac{-1}{y-2}\right) dy = 1\,dt$

$\boxed{\frac{1}{2}\ln y - \frac{1}{2}\ln(y-2) = t + C}$

or

$\ln\left(\frac{y}{y-2}\right) = 2t + C$

$\frac{y}{y-2} = Ce^{2t}$

... etc.

or

arctanh(...)

by ☐

3. Solve the initial value problem

$$\begin{cases} y' = \dfrac{3x^2}{2y-4}, \\ y(1) = 0. \end{cases}$$

What is the domain of the solution?

$\int (2y-4)\, dy = \int 3x^2\, dx$

$y^2 - 4y = x^3 + C$

$0 = 1 + C \rightarrow C = -1$

$y^2 - 4y + 4 = x^3 - 1 + 4$

$(y-2)^2 = x^3 + 3$

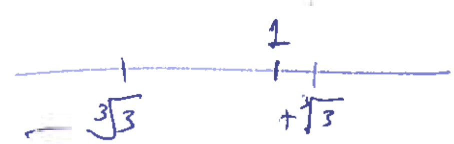
$y = 2 \pm \sqrt{x^3+3}$ $\underbrace{\text{only } - \text{ works.}}_{[2 \text{ pts}]}$

$\boxed{y = 2 - \sqrt{x^3+3}}$

[16 pts]

domain: $x^3 + 3 = 0$

$x = \pm\sqrt[3]{3}$



$\boxed{\text{domain} = (-\sqrt[3]{3}, \sqrt[3]{3})}$ ← [4 pts]

4. Solve the differential equation by making the change of variables $v = \frac{y}{x}$.

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

Be sure to give your solution in terms of x and y.

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{x^2 + xy + y^2}{x^2} = 1 + v + v^2$$

$$\rightarrow \quad \cancel{v} + x\frac{dv}{dx} = 1 + \cancel{v} + v^2$$

$$\frac{1}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Getting this correct [~15 pts]

$$\arctan(v) = \ln x + C$$

$$v = \tan(\ln(x) + C)$$ ← getting this is ok, but not done

$$\boxed{y = x\tan(\ln(x) + C)} \quad x \neq 0$$

[1 pt]

5. Solve the initial value problem

$$\begin{cases} y' = \dfrac{\cos t}{t^2} - \dfrac{2}{t}y, \\ y(\pi) = 0, \quad t > 0. \end{cases}$$

$y' + \frac{2}{t}y = \frac{\cos t}{t^2}$ linear!

$\mu = e^{2\ln t} = t^2$

$y = \frac{1}{t^2}\int \cancel{t^2}\frac{\cos t}{\cancel{t^2}}\,dt + \frac{c}{t^2}$

$= \frac{1}{t^2}(\sin t) + \frac{c}{t^2}$

$y = \frac{\sin t}{t^2} + \frac{c}{t^2}$ ← [15 pts]

$y(\pi) = \frac{0}{\pi} + \frac{c}{\pi^2} = 0 \Rightarrow c = 0$

[5 pts]

$\boxed{y = \frac{\sin t}{t^2}}$, $t > 0$