Name: M555: Differential Equations I (Su.19)
Good Problems 1
Sections 1.1-3, 2.1-2



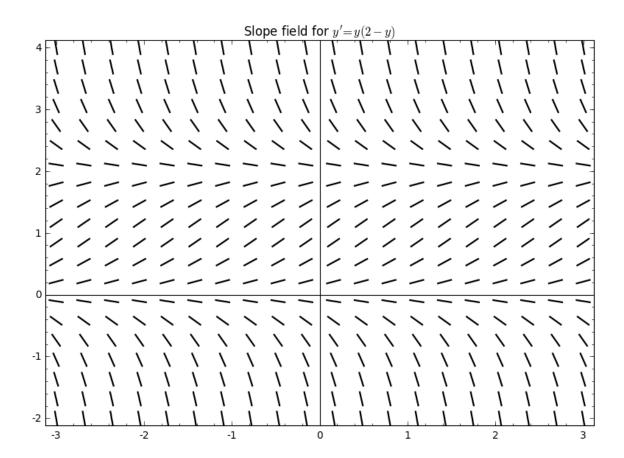
Instructions. Complete all problems, showing enough work. All work must be done on this paper. You may not use any notes or electronic devices. All you need is a pencil and your brain.

1. A field mouse population satisfies the differential equation

$$\frac{dp}{dt} = \frac{p}{2} - 450,$$

where p represents the population (in thousands) at time t (in years). If the initial population of field mice is p(0) = 450, find the time that the population will go extinct. Leave your answer in terms of a logarithm.

2. Given below is the slope field for the differential equation y' = y(2 - y). Plot and label the equilibrium solution(s), and sketch three non-equilibrium solution curves with distinctly different behavior.



Find the general solution of the DE. You do not need to solve for *y* explicitly.

3. Solve the initial value problem

$$\begin{cases} y' = \frac{3x^2}{2y - 4}, \\ y(1) = 0. \end{cases}$$

What is the domain of the solution?

4. Solve the differential equation by making the change of variables $v = \frac{y}{x}$.

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

Be sure to give your solution in terms of x and y.

5. Solve the initial value problem

$$\begin{cases} y' = \frac{\cos t}{t^2} - \frac{2}{t}y, \\ y(\pi) = 0, \quad t > 0. \end{cases}$$