Name:______________________________

M555: Differential Equations I (Su.19)

Good Problems 2

Sections 2.6, 2.4, 2.8

Instructions. Complete all problems, showing enough work. All work must be done on this paper. You may use your own hand-written notes, but you may not use any electronic devices.

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1. Consider the initial value problem,

$$\begin{cases} y' + y^3 = 0, \\ y(0) = y_0 \end{cases}$$

a.) [15 points] Find the solution of the IVP in terms of y_0.

b.) [10 points] Describe how the domain of definition of the solution function depends on the value of y_0. Be sure to include all possibilities.

2. Consider the differential equation

$$x^2 y^3 + x(1 + y^2) y' = 0.$$

a.) [5 points] Show that the DE is not exact as written.

b.) [5 points] Show that the DE becomes exact when using the integrating factor $\mu(x, y) = \frac{1}{xy^3}$.

c.) [15 points] Find the general solution to the DE. You do not need to solve for y explicitly.

3. Consider the differential equation

$$\begin{cases} \left(xy^2 + kx^2 y\right)\, dx + \left(x + y\right) x^2\, dy = 0, \\ y(1) = 1. \end{cases}$$

a.) [10 points] Find the value of k for which the differential equation is exact.

b.) [15 points] Using the value of k that you found in part *a*, solve the initial value problem. You do not need to solve for y explicitly.

4. [7 points] Make a linear change of variables to produce an equivalent differential equation with initial value $(0,0)$.

$$\begin{cases} y' = t^2 + y^2, \\ y(1) = 2. \end{cases}$$

5. [18 points] Consider the initial value problem,

$$\begin{cases} y' = ty + 1, \\ y(0) = 0. \end{cases}$$

Using the initial guess $\varphi_0(t) = 0$, perform Picard's iterative method to find $\varphi_1, \varphi_2, \varphi_3$, and φ_4.

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