**Name**: Key
**M555: Differential Equations I** (Su.19)
Good Problems 3
Sections 2.7, 3.1, 3.3

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**Instructions.** Complete all problems, showing enough work. All work must be done on this paper. You may use your own hand-written notes, but you may not use any electronic devices.

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**1.** [20 points] Consider the initial value problem

$$\begin{cases} y' = y(3 - ty), \\ y(0) = \frac{1}{2}. \end{cases}$$

Use Euler's method with a step size of $h = \frac{1}{2}$ to complete the table.

| $k$ | $t_k$ | $y_k$ |
|---|---|---|
| 0 | $0$ | $\boxed{1/2}$ |
| 1 | $1/2$ | $1/2 + (1/2 \cdot 3) \cdot 1/2 = 1/2 + 3/4 = \boxed{5/4}$ |
| 2 | $1$ | $5/4 + (5/4(3 - 1/2 \cdot 5/4))1/2 = 5/4 + 5/8(19/8) = 5/4 + 95/64 = \boxed{175/64}$ |
| 3 | $3/2$ | |
| 4 | $2$ | TOO MESSY to do by hand under pressure "! |

$$y_3: \frac{175}{64} + \left[\frac{175}{64} \cdot \left(3 - \frac{175}{64}\right)\right]\frac{1}{2} = \frac{175}{64} + \frac{175}{128} \cdot \frac{17}{64} = \frac{23400 + 2975}{8192} = \boxed{\frac{26375}{8192}}$$

**2.** [10 points] Find a differential equation whose general solution is

$$y(t) = C_1 e^{2t} + C_2 e^{-3t}.$$

$r = 2, -3$ so the char. eqn is:

$(r-2)(r+3) = r^2 + r - 6 = 0$

The SOLHCCDE is then

$$\boxed{y'' + y' - 6y = 0}$$

**3.** [30 points] Solve the initial value problem,

$$\begin{cases} y'' + 8y' - 9y = 0, \\ y(1) = 1, \quad y'(1) = 0. \end{cases}$$

Char. Eqn: $r^2 + 8r - 9 = 0$

$(r+9)(r-1) = 0$

$r = -9, 1$

Gen. Soln: $y = C_1 e^{-9t} + C_2 e^t$ $\rightarrow$ $y(1) = C_1 e^{-9} + C_2 e = 1$

$y' = -9C_1 e^{-9t} + C_2 e^t$ $\qquad$ $y'(1) = -9C_1 e^{-9} + C_2 e = 0$

$C_2 = \frac{9e^{-9}}{e} C_1 = 9e^{-10} C_1$

$C_1 e^{-9} + 9C_1 e^{-10} e = 1$

$C_1 = \frac{1}{10} e^9$

then $C_2 = \frac{9}{10e}$

and $\boxed{y(t) = \frac{1e^9}{10} e^{-9t} + \frac{9}{10e} e^t} = \boxed{\frac{1}{10} e^{-9(t-1)} + \frac{9}{10} e^{(t-1)}}$

**4.** [10 points] Write the number $e^{\ln(2)-\frac{\pi}{6}i}$ in the form $a+bi$.

$$e^{\ln 2-\frac{\pi}{6}i} = e^{\ln 2}\left(\cos(-\pi/6)+i\sin(-\pi/6)\right) = 2\left(\cos(\pi/6)-i\sin(\pi/6)\right)$$

$$= 2\left(\frac{\sqrt{3}}{2}-i\frac{1}{2}\right)$$

$$= \boxed{\sqrt{3}-i}$$

**5.** [30 points] Solve the initial value problem,

$$\begin{cases} y''-2y'+5y=0, \\ y(0)=2, \quad y'(0)=-3. \end{cases}$$

Char. Eq$\underline{\text{n}}$: $r^2-2r+5=0$

$r^2-2r+1=-4$

$(r-1)^2=-4$

$r=1\pm 2i$

Gen. Sol$\underline{\text{n}}$: $y=e^t\left(c_1\cos(2t)+c_2\sin(2t)\right)$

$y'=e^t\left(c_1\cos(2t)+c_2\sin(2t)-2c_1\sin(2t)+2c_2\cos(2t)\right)$

$y(0)=c_1=2$

$y'(0)=c_1+2c_2=-3 \to c_2=\frac{-3-2}{2}=\frac{-5}{2}$

So, $\boxed{y(t)=e^t\left(2\cos(2t)-\frac{5}{2}\sin(2t)\right)}$

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