M555: Differential Equations I (Su.19)

Good Problems 5 Sections 5.1 and 5.2

Due: Wednesday, 3 July 2019 at 9:50 am



Instructions. Complete all problems on this paper. You may use any resources that you'd like, but be sure to show enough work.

1. [10 points] Determine the radius of convergence of the power series,

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^2 (x+2)^n}{3^n}.$$

2. [10 points] use your favorite method to find the Taylor series for the function $f(x) = x^2$ about the point $x_0 = -1$.

3. [15 points] Use your favorite method to find the Taylor series for $f(x) = \frac{1}{1-x}$ about $x_0 = 0$. What is the interval of convergence?

4. [15 points] Use your favorite method to find the Taylor series for $f(x) = \ln(x-1)$ about $x_0 = 2$. What is the interval of convergence?

5. [25 points] Find power series solutions to the second order differential equation

$$y'' - xy' - y = 0$$

centered about the point $x_0 = 0$.

6. [25 points] Find a power series solution to the initial value problem,

$$\begin{cases} xy'' + y' + xy = 0, \\ y(1) = 1, \\ y'(1) = -2. \end{cases}$$

