Name:_____

M555: Differential Equations I (Su.19)
Good Problems 6
Sections 5.2 – 5.5

Instructions. Complete all problems, showing enough work. All work must be done on this paper. You may use your own hand-written notes, but you may not use any electronic devices.

1. [25 points] Consider the *Hermite Equation*,

$$y'' - 2xy' + \lambda y = 0,$$

where λ is a real constant. Find and clearly identify the recurrence relation for the solution centered at $x_0 = 0$. You do **not** need to solve for the coefficients.

2. [30 points] Consider the differential equation

$$y'' + (\sin x)\, y = 0.$$

Use your favorite method to find the first three non-zero terms of each of the power series solutions centered at $x_0 = 0$. You do **not** need to write the solution in Σ-notation.

3. [15 points] Find and classify (as regular or irregular) all singular points of the differential equation.

$$x^2(1-x)\,y'' + (x-2)\,y' - 3xy = 0.$$

Do **not** solve the equation.

4. [30 points] Consider the differential equation

$$x y'' + y = 0.$$

Show that $x_0 = 0$ is a regular singular point; find the exponents at $x_0 = 0$; and find the first four non-zero terms of the series solution corresponding to the larger exponent.

scratch page