Sections 5.5, 6.1–6.3



Instructions. Complete all problems, showing enough work. All work must be done on this paper. You may use your own hand-written notes, but you may not use any electronic devices.

1. [30 points] Consider the second order differential equation

$$x^2y'' + xy' + (x-2)y = 0.$$

(a.) Show that $x_0 = 0$ is a regular singular point; (b.) Determine the indicial equation and the exponents at the singularity; and (c.) Find the series solution (x > 0) corresponding to the larger exponent.

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$$\mathcal{L}\big\{t\sin(2t)\big\}.$$

You must use the definition to receive credit. Be sure to treat any improper integrals properly.

3. [20 points] Find the inverse Laplace transform, $\mathcal{L}^{-1}{F(s)}$, where

$$F(s) = \frac{2s+2}{s(s^2+4s+5)}.$$

4. [30 points] Use the method of Laplace transforms to solve the initial value problem

$$\begin{cases} y'' + 16y = \cos(2t), \\ y(0) = 1, \\ y'(0) = 0. \end{cases}$$

