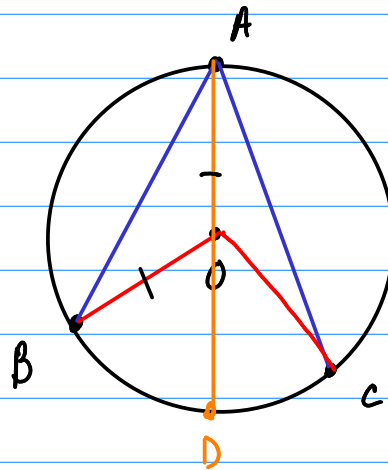


§1.3:

Star Trek Lemma

We will prove for the case $\angle BAC$ is acute and O is inside the angle.



$$\angle BAC = \frac{1}{2} \angle BOC$$

LHS: $\overline{AO} = \overline{BO} = \overline{OD}$ because they are all radii of the circle.

Then $\triangle BAO$ is isosceles.

So, $\angle OBA = \angle BAO$

$\angle AOD$ is a straight line, and $\angle AOD$ and $\triangle BOA$ share the common angle $\angle BOA$.

Therefore $\angle BOD = \angle OBA + \angle BAO$

$$\angle BOD = 2 \angle BAO$$

Similarly on RHS,

$$\angle COD = 2 \angle CAO$$

and

$$\angle BOC = \angle BOD + \angle DOC$$

$$= 2 \angle BAO + 2 \angle OAC$$

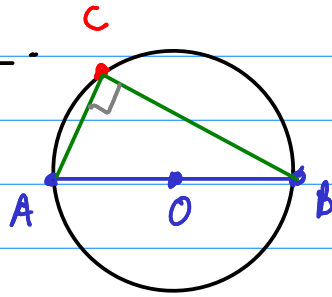
$$= 2 (\angle BAO + \angle OAC)$$

$$= 2 \angle BAC$$

□

RE. Read the other cases.

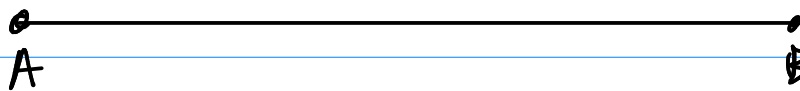
Corollary -



$\angle ACB$ is 90° by Star Trek.
So $\triangle ABC$ is right.

§1.4 - Some basic constructions

Ex. 1. Given a line segment, divide it into n equal segments.
 $n=4$.



Ex. 1/2. Copy an angle

Ex. 2. a.) bisect a line segment
b.) bisect an angle

Ex. 3. Perp. bisector of a chord

Ex. 4. Construct the circumcircle of a given triangle.

Ex. 5. Tangent line to a given circle ^{through} a given point
not on the circle.