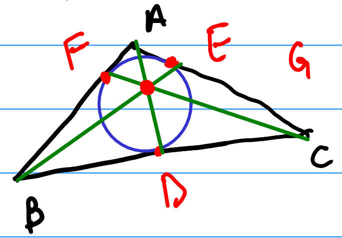
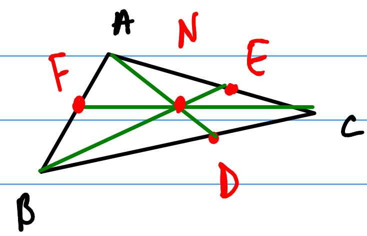
M621 9/26/18

Gergonne Pt

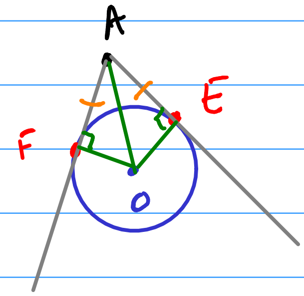


Nagel Pt



$\underbrace{\overline{AB} + \overline{BD} = s = \overline{DC} + \overline{CA}}$, ... etc.

Proof of Gergonne: "Zoom In" on vertex A



Since AF and AE are tangents to the circle through A, then $\overline{AF} = \overline{EA}$

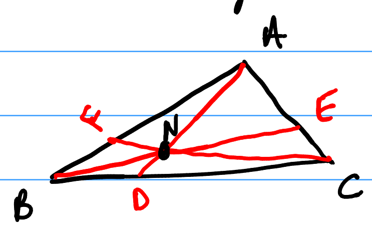
Similarly, $\overline{FB} = \overline{BD}$ and $\overline{DC} = \overline{CE}$.

Then,

$$\frac{\overline{AF}}{\overline{FB}} \cdot \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} = 1$$

The lines are concurrent by Ceva's Thm. ∎

Nagel's Proof:



$\overline{AF} + \overline{FB} + \overline{BD} = s$

and $\overline{FB} + \overline{BD} + \overline{DC} = s$

therefore,

$\overline{AF} + \cancel{\overline{FB}} + \cancel{\overline{BD}} = s = \cancel{\overline{FB}} + \cancel{\overline{BD}} + \overline{DC}$

then $\overline{AF} = \overline{DC}$

Similarly, $s = \overline{BD} + \overline{DC} + \overline{CE} = \overline{DC} + \overline{CE} + \overline{EA} \Rightarrow \overline{BD} = \overline{EA}$

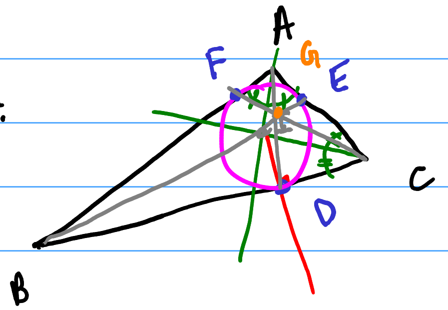
$s = \overline{CE} + \overline{EA} + \overline{AF} = \overline{EA} + \overline{AF} + \overline{FB} \Rightarrow \overline{CE} = \overline{FB}$

Then,

$$\frac{\overline{AF}}{\overline{FB}} \cdot \frac{\overline{BD}}{\overline{DC}} \cdot \frac{\overline{CE}}{\overline{EA}} = 1 \quad ∎$$

Constructing the Gergonne and Nagel pts:

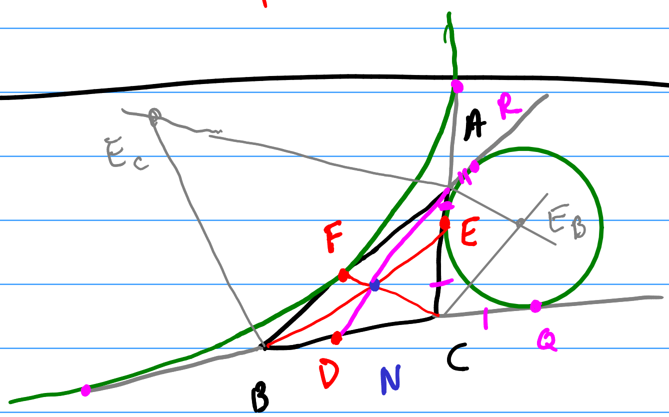
Gergonne:



Step 1: Inscribe a circle.

Step 2: Connect vertices to pts of tangency.

Nagel:



$\overline{CE} = \overline{CQ}$

$\overline{EA} = \overline{AR}$ ←

$\overline{BQ} = \overline{BR}$

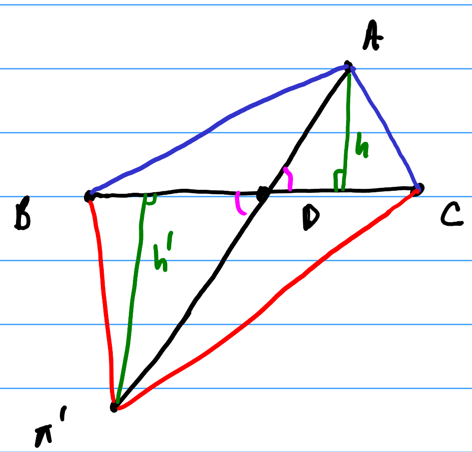
but $\overline{BQ} = \overline{BC} + \overline{CQ} = \overline{BC} + \overline{CE}$

and $\overline{BR} = \overline{BA} + \overline{AR} = \overline{BA} + \overline{AE}$

if $\overline{BC} + \overline{CE} = \overline{BA} + \overline{AE}$, then each must equal half the perimeter.

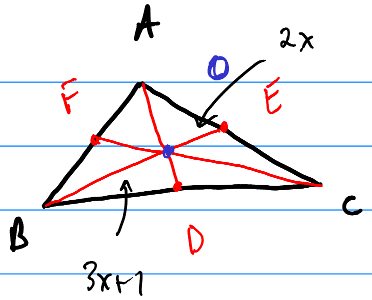
since $2s = \overline{BC} + \overline{CE} + \overline{EA} + \overline{AB}$

2.3.3:



$$\frac{\Delta \overline{ABC}}{\Delta A'BC} = \frac{\overline{AD}}{\overline{A'D}}$$

Ex. 2.3.1.



$\overline{BO} = 2\,\overline{OE}$, etc.

Proof ?