M621

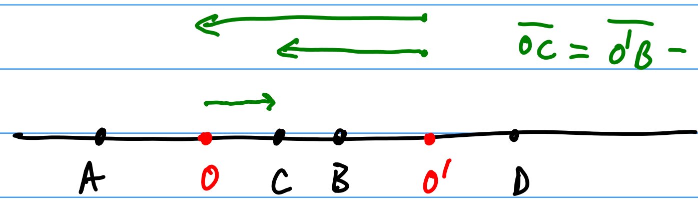
Hw $(AB, CD) = -1$ O midpt of AB : $\overline{AO} = \overline{OB}$

O' midpt of CD $\overline{CO'} = \overline{O'D}$

want: $\overline{O'C}^2 + \overline{OB}^2 = \overline{OO'}^2$

$\overline{OC} = \overline{O'B} - \overline{O'O}$

Picture:



$\overline{O'C}$ $\overline{O'D}$ $\overline{O'D} = -\overline{O'C}$

Thm 2.6.4:
$$\boxed{\overline{OB}^2 = \overline{OC}\cdot\overline{OD}}$$
$$+\ \overline{O'C}^2 = \overline{O'A}\cdot\overline{O'B}$$

$$\overline{OB}^2 + \overline{O'C}^2 = \boxed{\underbrace{\overline{OC}\cdot\overline{OD}}_{\substack{O' \\ \text{origin}}}} + \underbrace{\overline{O'A}\cdot\overline{O'B}}_{\substack{O \\ \text{origin}}}$$

$$= (\overline{O'C} - \overline{O'O})(\underbrace{\overline{O'D}}_{-\overline{O'C}} - \overline{O'O}) = (-\overline{O'O} + \overline{O'C})(-\overline{O'O} - \overline{O'C})$$

$(a+b)(a-b)$

$$= \boxed{\overline{O'O}^2 - \overline{O'C}^2}$$

... eventually ...

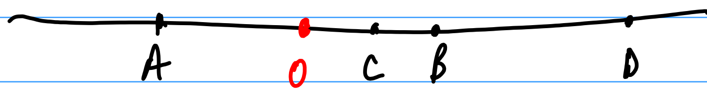
$$\overline{OB}^2 + \overline{O'C}^2 = \overline{O'O}^2 - \overline{O'C}^2 + \overline{O'O}^2 - \overline{OB}^2$$

$$2\,\overline{OB}^2 + 2\overline{O'C}^2 = 2\,\overline{O'O}^2$$

So, $\overline{OB}^2 + \overline{O'C}^2 = \overline{O'O}^2$ ■ :)

Thm. 2.6.4 $\quad (AB, CD) = -1 \quad O$ is the midpoint of AB.

show: $\overline{OB}^2 = \overline{OC} \cdot \overline{OD}$.



$$(AB, CD) = -1 = \frac{\overline{AC}}{\overline{CB}} \cdot \frac{\overline{DB}}{\overline{AD}}$$

$$\Rightarrow \frac{\overline{AC}}{\overline{CB}} = -\frac{\overline{AD}}{\overline{DB}}$$

$$\Rightarrow \overline{AC} \cdot \overline{DB} = -\overline{AD} \cdot \overline{CB}$$

$$(\overline{OC} - \overline{OA})(\overline{OB} - \overline{OD}) = -(\overline{OD} - \overline{OA})(\overline{OB} - \overline{OC})$$

$$+(\overline{OB}) \qquad\qquad +(\overline{OB})$$

$$\Rightarrow \cancel{\overline{OB} \cdot \overline{OC}} + \overline{OB}^2 - \overline{OC} \cdot \overline{OD} - \cancel{\overline{OB} \cdot \overline{OD}} = +\left(\cancel{-\overline{OD} \cdot \overline{OB}} + \overline{OD} \cdot \overline{OC} - \overline{OB}^2 + \cancel{\overline{OB} \cdot \overline{OC}} \right)$$

$$\overline{OB}^2 - \overline{OC} \cdot \overline{OD} = -\overline{OB}^2 + \overline{OD} \cdot \overline{OC}$$

$$2\,\overline{OB}^2 = 2\,\overline{OC} \cdot \overline{OD} \quad \ldots \quad \checkmark$$

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§2.7 - ~~orthogonal~~ circles

Defn.

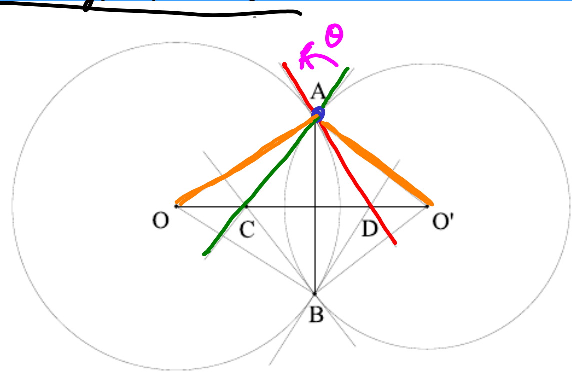
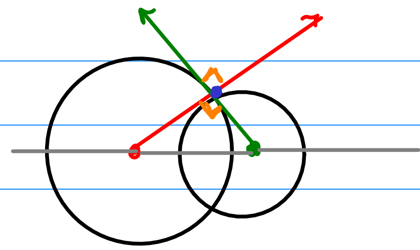


Figure 2.7.1

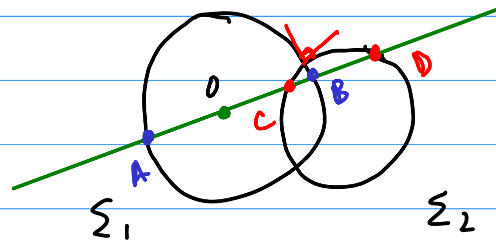
The angle between the circles Σ_1 and Σ_2 is defined to be the angle between the respectively tan. lines at the intersection.

Defn. Σ_1 and Σ_2 are orthogonal if $\theta = \pi/2 = 90°$.

If $\Sigma_1 \perp \Sigma_2$, then the tan. lines pass through the opposite centers:



<u>Thm.</u> If Σ_1 and Σ_2 are orthogonal, then any diameter of one circle is cut harmonically by the other circle.

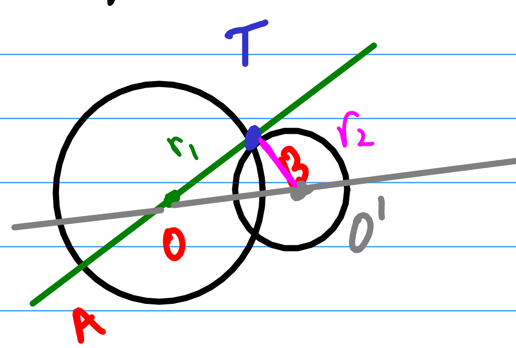


$(AB, CD) = -1$

From the proof: $\overline{OB}^2 = \overline{OC} \cdot \overline{OD}$

so, $r_1^2 = \overline{OC} \cdot \overline{OD}$

If the diameter is tangent to the second circle at T,



<u>Pythagorean Thm</u>: $\overline{OO'}^2 = r_1^2 + r_2^2$

$\overline{OO'}^2 = \overline{OT}^2 + \overline{O'T}^2$

<u>Q.</u> Given a circle and 2 pts not on the circle, construct the circle orthogonal to the given ones and passing through both pts.

Picture: