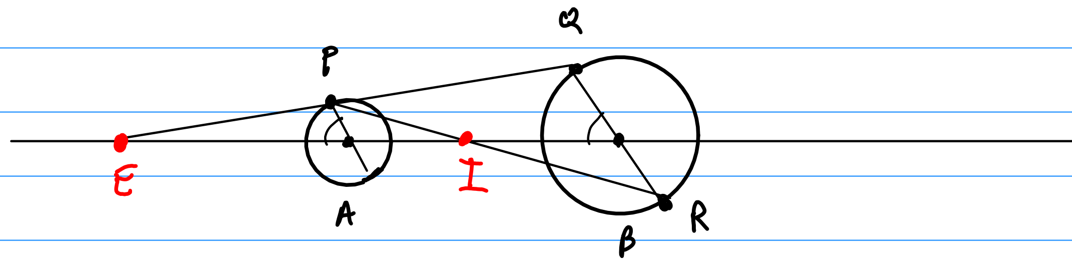
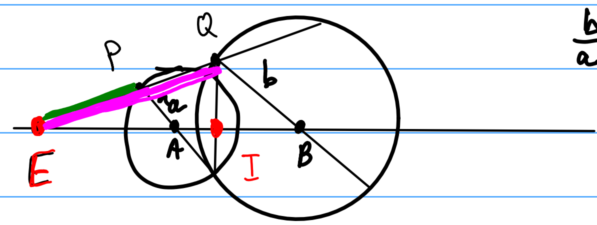
M621 29 Oct 18

## §3.2 – Applications of Homothetics

**Def'n.** Given two non-concentric circles $A(a)$ and $B(b)$. The interior and exterior centers of similitude, $I$ and $E$, are the points on $AB$ that divide $AB$ in the same ratio.




or




$$\frac{b}{a}\overline{EP} = \overline{EQ}$$

$H_{(E,k)}(P) = Q$ means $\overline{EQ} = k\,\overline{EP}$.

**Thm.** Any two non-concentric circles $A(a)$ and $B(b)$ are homothetic images of one another. $B(b)$ is the image of $A(a)$ under

$$H_{(E,\, b/a)}$$

and

$$H_{(I,\, -b/a)}$$

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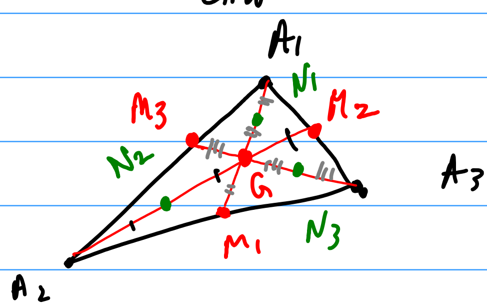
**Thm.** Given $\Delta A_1A_2A_3$. Let $M_1, M_2, M_3$ be the midpoints of the sides (opposite), and let $G$ be the centroid. Then

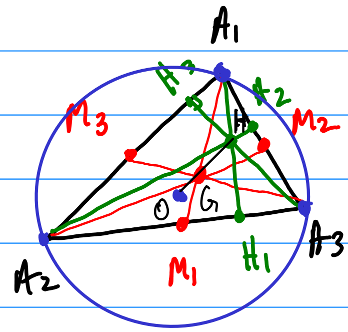
$$\frac{\overline{A_iG}}{\overline{GM_i}} = 2$$




$N_2$ = midpoint of $\overline{A_2G}$

Thm. The orthocenter $H$ of triangle $A_1A_2A_3$, the circumcenter $O$, and the centroid $G$ are collinear and

$$\overline{HG} = 2\,\overline{GO}.$$




Defn. The line $\overline{HGO}$ is the Euler line of $\Delta A_1A_2A_3$.

Thm. Nine Point Circle Theorem. In any triangle $A_1A_2A_3$ w/ $M_1, M_2, M_3$, $H_1, H_2, H_3$, and $N_1, N_2, N_3$ being the midpoints of $\overline{A_iH}$. The nine points lie on a circle w/ center $N$ the midpoint of $\overline{HO}$ and radius half that of the circumcircle's radius.