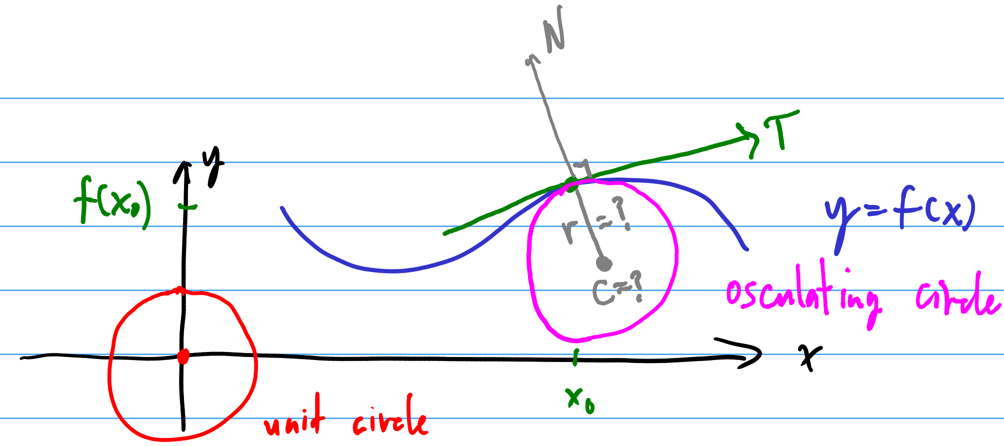
GP 19.



$T_{\vec{c}} \circ H_{(0,r)}$

↑ ↑ ?

GP 20.

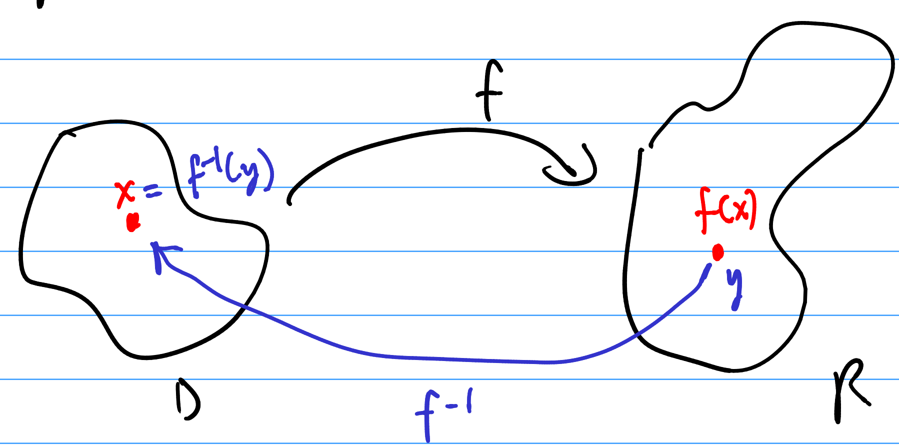
$\begin{cases} f, g \text{ are } \underline{\text{isometries}} \\ f(0) = 0 \\ g(0) = 0 \end{cases}$ $\quad f \circ g$ is an isometry?

$f \circ g\,(0) = f\big(g(0)\big) \; \ldots$

$\overline{PQ} = \overline{f(PQ)}$

$\overline{PQ} = \overline{g(PQ)}$

$f \circ g\,(PQ)$?



$f^{-1} \circ f(x) = f^{-1}(y)$

$x = f^{-1}(y)$

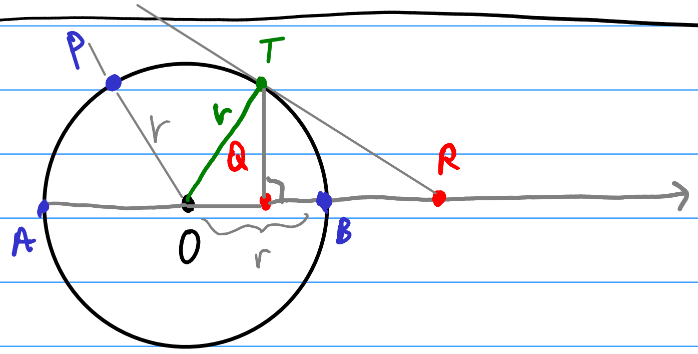
M621 31 Oct '18

§3.4 – Inversions in Circles

Def'n The *inversion* in circle $O(r)$ is the transformation that takes a pt P not equal to the center to the point Q on OP satisfying

$$(\overline{OP})(\overline{OQ}) = r^2$$

$I_{O(r)} = I_{(O,r)}$



Thm. If P lies on the circle, then $I_{O(r)}(P) = P$.

Thm. Points inside the circle map to points outside the circle, and vice versa.

$I_{O(r)}(Q) = R$ and $I_{O(r)}(R) = Q$

Thm. Inversion is an *involution*: $f \circ f = id$ or $I_{O(r)} \circ I_{O(r)}(P) = P$.

Thm. Let R be outside $O(r)$ and T be a point of tangency: TR is tangent to $O(r)$. Then the image $I_{O(r)}(R) = Q$ lies on the line perpendicular to OR, passing through T.

"Proof:" Ch 2. $(\overline{OQ})(\overline{OR}) = r^2 = \overline{OT}^2$.

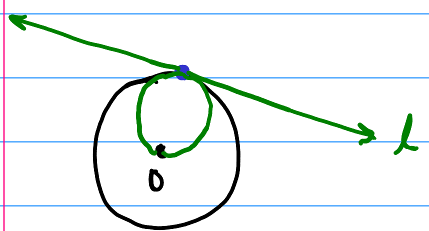
Cor. Let A, B be the points of intersection of OR w/ the circle $O(r)$. Then $(AB, QR) = -1$.

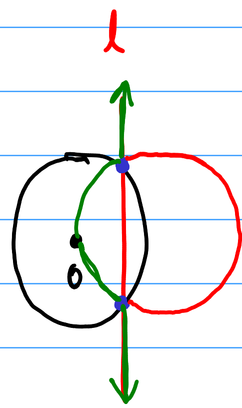
or $\dfrac{\overline{AQ}}{\overline{QB}} \cdot \dfrac{\overline{RB}}{\overline{AR}} = -1$

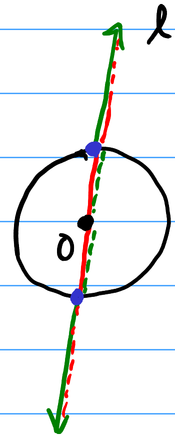
Construction. Given a circle $O(r)$ and any pt, we can construct the image under the inversion using a method of the harmonic conjugate.

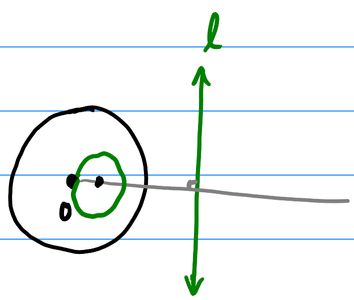
Thm. The center O maps to ∞, and vice versa, under $I_{O(r)}$.

Thm. A line ℓ maps to:

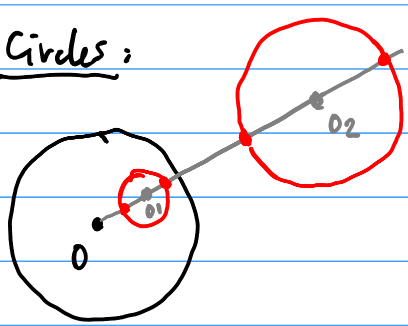


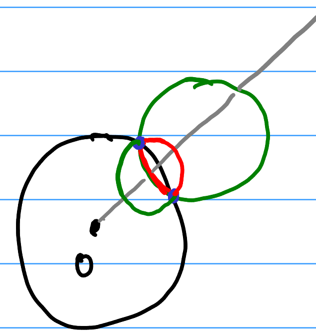






Thm. Circles:





Note: $I_{O(r)}(O_1) \neq O_2$!