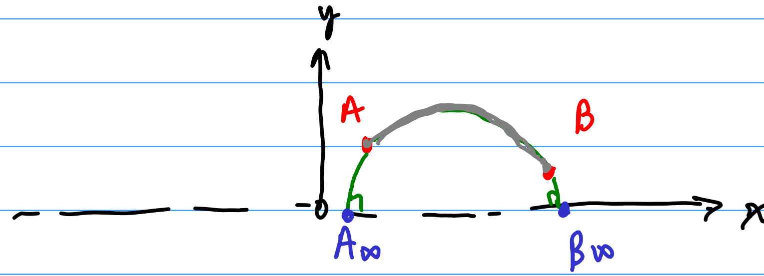
M621 19 Nov'18

Distance in $\mathbb{H}$.




$A_\infty = (1,0)$

$B_\infty = (4,0)$

$$d_{\mathbb{H}}(A,B) = \ln\left(\frac{\overline{AB_\infty}\cdot\overline{BA_\infty}}{\overline{AA_\infty}\cdot\overline{BB_\infty}}\right)$$

$A = (1.5, 3)$

$$\overline{AB_\infty} = \sqrt{(1.5-4)^2+(3-0)^2}$$

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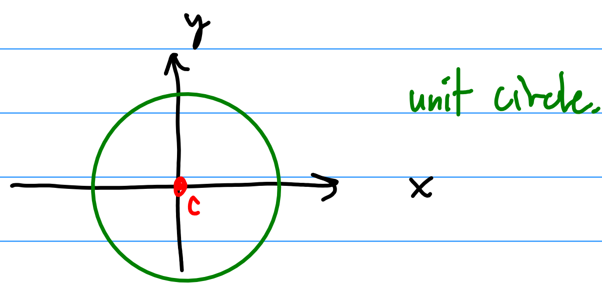
## §4.5 Taxi Cab Geometry

Euclidean distance: $0=(0,0)$, $\bar{x}=(x,y)$

$$d(0,\bar{x}) = \sqrt{x^2+y^2} = (x^2+y^2)^{1/2} = \|\bar{x}\|_2$$

$d(0,\bar{x})=1$ the locus of all $\bar{x}$ satisfying $d(0,\bar{x})=1$



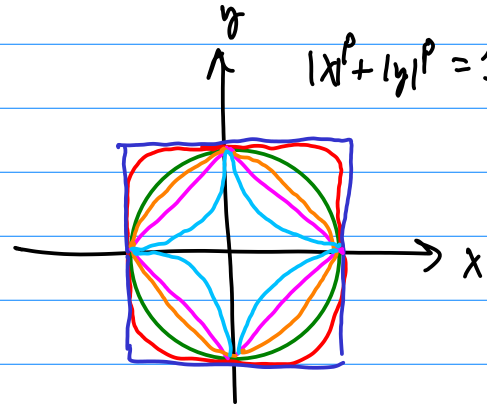

Let $p>0$. Then the p-norm of $\bar{x}$ is the number

$$\|\bar{x}\|_p = d_p(0,\bar{x}) = (|x|^p+|y|^p)^{1/p}$$

Q. what happens to the unit circle $d_p(0,\bar{x})=1$ as $p$ varies.

The $\infty$-norm is $d_\infty(0,\bar{x}) = \lim_{p\to\infty} d_p(0,\bar{x}) = \max(|x|,|y|)$

Unit circles in p-norms:




$\infty > p > 2$

$p=\infty$

$p>2$

$1<p<2$

$p=1$

$0<p<1$

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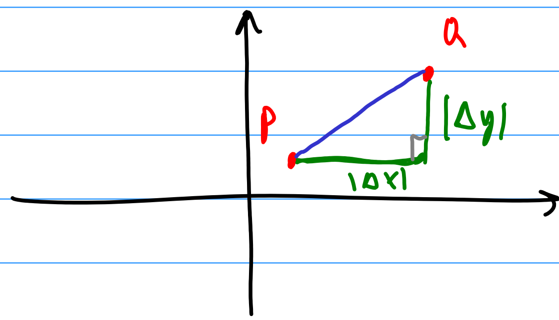
Taxi Cab distance is $p=1$:

$$d_T(P,Q) = |x_2-x_1| + |y_2-y_1|$$

$P(x_1,y_1)$
$Q(x_2,y_2)$




$P(1,2)$
$Q(4,5)$

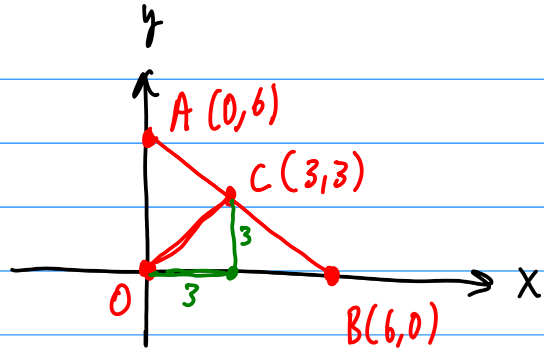
$$d_T(P,Q) = |4-1| + |5-2| = 3+3=6$$

$$d(P,Q) = \sqrt{3^2+3^2} = \sqrt{2\cdot 3^2} = 3\sqrt{2}$$

The angles in taxi cab geometry are the same as the Euclidean angles.

Thm. The S-A-S congruence Postulate fails in taxi cab geom. All of the others still hold.

Ex.




$\Delta AOB$ : $\measuredangle AOB = \pi/2 = 90°$
$\overline{OA} = 6$
$\overline{OB} = 6$

$\Delta OCB$ : $\measuredangle OCB = \pi/2 = 90°$
$\overline{OC} = d_T(O,C) = 6$
$d_T(C,B) = 6$

$\Delta AOB$ and $\Delta OCB$ satisfy SAS hypothesis. However,

$$\left.\begin{aligned} d_T(A,B) &= 12 \\ d_T(O,B) &= 6 \end{aligned}\right\} \neq$$

These $\Delta$s are not congruent.

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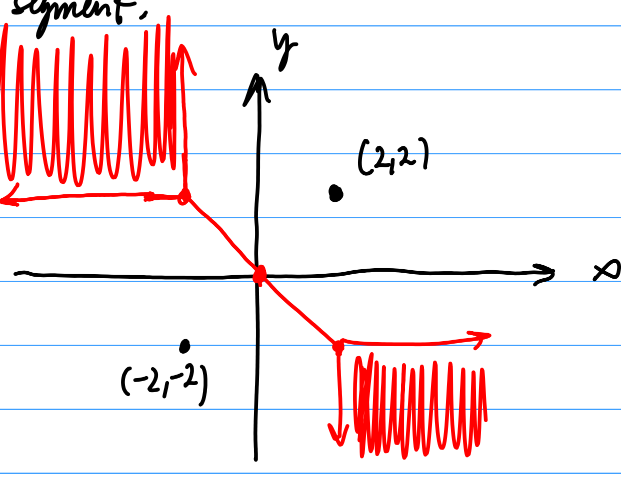
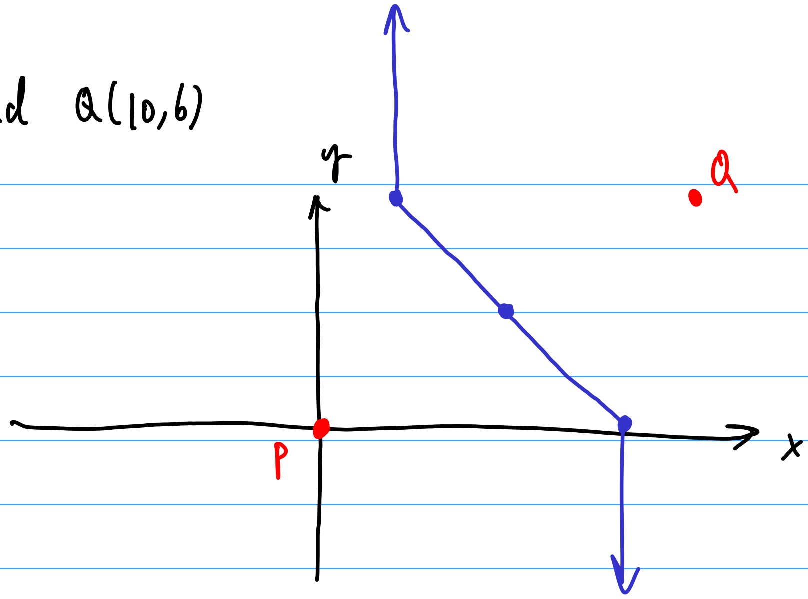
Ex. Find the locus of pts equidistant from (2,2) and (-2,-2)

In Euclidean geometry this would give a line: perpendicular bisector of the segment.

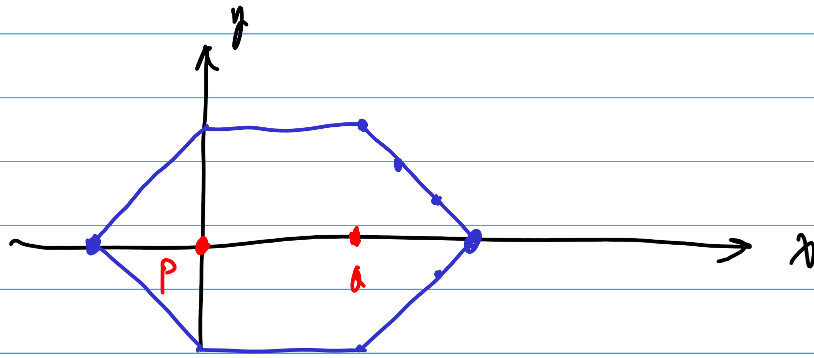


Ex. P(0,0) and Q(10,6)




Ex. Ellipse w/ Focii P(0,0) and Q(4,0).

$d_T(P,X) + d_T(Q,X) = 10$ Find all X

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