Name:______________________________
M511: Linear Algebra (Spring 2018)
Instructor: Justin Ryan
Unit II Exam: Chapter 5 (In Class)

Instructions. *Read and follow all instructions. You may not use a calculator or any other electronic device. You may use a two-sided 8.5" × 11" page of your own hand-written notes.*

Part I. True/False [2 points each] *Neatly write* **T** *on the line if the statement is always true, and* **F** *otherwise. In the space provided below the statement, give sufficient explanation of your answer.*

______ **1.a.** Let $A \in \mathbb{R}^{m\times n}$. The system $A\mathbf{x} = \mathbf{b}$ has a unique least squares solution.

______ **1.b.** Let $Q \in \mathbb{R}^{n\times n}$. If the columns of Q are orthonormal, then $Q^T Q = I$.

______ **1.c.** Let $A \in \mathbb{R}^{m\times n}$. If $\operatorname{rank}(A) = n$, then $\operatorname{Null}(A^T) = \{\mathbf{0}\}$.

______ **1.d.** Let $G \in \mathbb{R}^{n\times n}$ such that $\det(G) = 0$. Then the formula $\langle \mathbf{x}, \mathbf{y}\rangle = \mathbf{x}^T G\mathbf{y}$ defines an inner product on the vector space $\mathbb{R}^n$.

______ **1.e.** Let $\mathscr{B} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ be an orthonormal basis for the inner product space $(V, \langle\ ,\ \rangle)$. The matrix representation of $\langle\ ,\ \rangle$ with respect to the basis $\mathscr{B}$ is the identity matrix.

Part II. Written Problems [12.5 points each] *Complete all problems, showing enough work in the space provided.*

2. Let $(V, \langle\ ,\ \rangle)$ be a finite-dimensional inner product space. Prove the triangle inequality:

$$\textit{For any } \mathbf{x}, \mathbf{y} \in V,\ \|\mathbf{x}+\mathbf{y}\| \leq \|\mathbf{x}\| + \|\mathbf{y}\|.$$

When does equality hold?

3. Consider the vector space $S = \text{span}\left\{e^t, e^{2t}, e^{3t}\right\}$ together with the inner product

$$\langle f, g\rangle = \int_0^{\ln 2} f(t)g(t)\,dt.$$

Find the matrix representation of this inner product with respect to the ordered basis $\left\{e^t, e^{2t}, e^{3t}\right\}$.

4. Use the method of least squares to find the equation of the best fit line to the date points.

x	-2	-1	0	1	2
y	-1	1	4	6	9

5. Consider the subset S of $(\mathbb{R}^4, \cdot\,)$ spanned by the ordered basis

$$\mathscr{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right\}.$$

Apply the Gram-Schmidt orthogonalization process to $\mathscr{B}$ to find an orthonormal basis of S.