Name:_____

M511: Linear Algebra (Summer 2018)

Good Problems 1: Chapter 1

WICHITA STATE UNIVERSITY

Instructions *Complete all problems on this paper, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. Solve the system of equations by using Gaussian or Gauss-Jordan elimination.

$$\begin{cases} x_2 + x_3 + x_4 & = 0 \\ 3x_1 + 3x_3 - 4x_4 & = 7 \\ x_1 + x_2 + x_3 + 2x_4 & = 6 \\ 2x_1 + 3x_2 + x_3 + 3x_4 & = 6 \end{cases}$$

2. Let A be a matrix of the form

$$A = \begin{pmatrix} \alpha & \beta \\ 2\alpha & 2\beta \end{pmatrix}$$

where α and β are fixed constants not both equal to 0.

a.) Explain why the system $A\mathbf{x} = (3,\ 1)^T$ must be inconsistent.

b.) how can one choose a nonzero vector $\mathbf{b}$ so that the system $A\mathbf{x} = \mathbf{b}$ will be consistent? Explain.

3. Let A be a 3×3 matrix and let $\mathbf{b} = 3\mathbf{a}_1 + \mathbf{a}_2 + 4\mathbf{a}_3$. Will the system $A\mathbf{x} = \mathbf{b}$ be consistent? Explain.

4. Let A be a 3×3 matrix and suppose that $\mathbf{a}_1 - 3\mathbf{a}_2 + 2\mathbf{a}_3 = \mathbf{0}$. Is A nonsingular? Explain.

5. Let E and F be $n \times n$ elementary matrices and let $C = EF$. Is C nonsingular? Explain.

6. Let A and B be 10×10 matrices that are partitioned into submatrices

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

a.) If $A_{11} \in \mathbb{R}^{6\times 5}$ and $B_{11} \in \mathbb{R}^{k\times r}$, what conditions, if any, must k and r satisfy in order to make the block multiplication of A times B possible?

b.) Assuming that the block multiplication is possible, how would the $(2,2)$ block of the product be determined?