Name:______________________________

M511: Linear Algebra (Summer 2018)

Good Problems 3: Sections 3.1–3.4

Instructions *Complete all problems on this paper, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. In $\mathbb{R}^3$ let $\mathbf{x}_1$ and $\mathbf{x}_2$ be linearly independent vectors and let $\mathbf{x}_3 = \mathbf{0}$. Are $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ linearly independent? Prove your answer.

2. Consider the subset $S = \{\mathbf{x} \in \mathbb{R}^2 \mid x_1 \cdot x_2 = 0\}$. Is S a subspace of $\mathbb{R}^2$? Prove your answer.

3. Consider the vectors

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}, \quad \mathbf{x}_3 = \begin{pmatrix} 1 \\ 5 \\ 5 \end{pmatrix}, \quad \mathbf{x}_4 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

a.) Are $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$, and $\mathbf{x}_4$ linearly independent in $\mathbb{R}^3$? Explain.

b.) Do $\mathbf{x}_1, \mathbf{x}_2$ span $\mathbb{R}^3$? Explain.

c.) Do $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ span $\mathbb{R}^3$? Are they linearly independent? Do they form a basis for $\mathbb{R}^3$? Explain.

d.) Do $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_4$ span $\mathbb{R}^3$? Are they linearly independent? Do they form a basis for $\mathbb{R}^3$? Explain.

4. Let S be the set of all symmetric 2×2 matrices with real entries. (*a*) Show that S is a subspace of $\mathbb{R}^{2\times 2}$; (*b*) Find a basis for S.

5. Let $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ be linearly independent vectors in $\mathbb{R}^4$ and let $A \in \mathbb{R}^{4\times 4}$ be nonsingular. Prove that if $\mathbf{y}_1 = A\mathbf{x}_1$, $\mathbf{y}_2 = A\mathbf{x}_2$, $\mathbf{y}_3 = A\mathbf{x}_3$, then $\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3$ are linearly independent.