Name:________________________________

M511: Linear Algebra (Summer 2018)

Good Problems 5: Chapter 5

WICHITA STATE UNIVERSITY

Instructions *Complete all problems on this paper, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. Let

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} -2 \\ 1 \\ 2 \\ 0 \end{pmatrix}.$$

a) Find the vector projection $\mathbf{p}$ of $\mathbf{x}$ onto $\mathbf{y}$.

b) Verify that $\mathbf{x} - \mathbf{p}$ is orthogonal to $\mathbf{p}$.

c) Verify that the Pythagorean Law holds for $\mathbf{x}, \mathbf{p}$, and $\mathbf{x} - \mathbf{p}$.

2. Prove the *Triangle Inequality* in $\mathbb{R}^n$:

$$\|\mathbf{x}_1 + \mathbf{x}_2\| \le \|\mathbf{x}_1\| + \|\mathbf{x}_2\|$$

for all $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^n$.

3. Prove the *Cauchy-Schwarz-Bunyachevsky Inequality* in $\mathbb{R}^n$:

$$|\mathbf{x}^T \mathbf{y}| \le \|\mathbf{x}\| \, \|\mathbf{y}\|$$

for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$.

4. Use trigonometry in $\mathbb{R}^n$ to prove the formula

$$\mathbf{x}^T\mathbf{y} = \|\mathbf{x}\|\,\|\mathbf{y}\|\,\cos\theta$$

where θ is the smallest angle between $\mathbf{x}$ and $\mathbf{y}$.

5. Prove the *Parallelogram Law* in $\mathbb{R}^n$:

$$\|\mathbf{x}+\mathbf{y}\|^2 + \|\mathbf{x}-\mathbf{y}\|^2 = 2\|\mathbf{x}\|^2 + 2\|\mathbf{y}\|^2$$

for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$.

6. Apply the Gram-Schmidt algorithm to find an orthonormal basis for the column space of the matrix

$$A = \begin{pmatrix} 1 & -3 & -5 \\ 1 & 1 & -2 \\ 1 & -3 & 1 \\ 1 & 1 & 4 \end{pmatrix}$$

7. Find the best least squares fit line to the data.

x	-1	1	2
y	1	3	3

8. Consider the inner product on $\mathbb{P}_n$ defined by

$$\langle p, q\rangle = \int_{-1}^{1} p(x)q(x)\,dx.$$

Starting with the standard basis $\{1, x, x^2, x^3\}$ of $\mathbb{P}_4$, use the Gram-Schmidt algorithm to find an orthonormal basis of $\mathbb{P}_4$ with respect to this inner product.