Name:________________________________
M511: Linear Algebra (Summer 2018)
Good Problems 6: Chapter 6

Wichita State University

Instructions *Complete all problems on this paper, showing enough work. A selection of problems will be graded based on the organization and clarity of the work shown in addition to the final solution (provided one exists).*

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1. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \\ 1 & 2 & -2 \end{pmatrix}.$$

a.) Find all eigenvalues and eigenspaces of A.

b.) Factor A into a product $A = XDX^{-1}$, where D is a diagonal matrix.

c.) Use your answer to part (*b*) to compute A^7.

2. Let A be a nonsingular $n \times n$ matrix, and let λ be an eigenvalue of A. *a.*) Show that $\lambda \neq 0$; *b.*) Show that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

3. Let A be a matrix whose columns all add up to a fixed constant δ. Show that δ is an eigenvalue of A.

4. Consider the matrices

$$A = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}, \quad \text{and} \quad \mathbf{Y}_0 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

Compute e^{tA} and use it to compute the solution of the initial value problem $\mathbf{Y}'(t) = A\mathbf{Y}$, $\mathbf{Y}(0) = \mathbf{Y}_0$.

5. Consider the real vector space $\mathbb{C} = \left\{z = x + iy \mid x, y \in \mathbb{R}\right\}$, and the linear transformation $L : \mathbb{C} \to \mathbb{C}$ defined by $L(z) = iz$. What do you think the eigenvalues and eigenvectors of L should be? Check your answer by writing the matrix representation of L with respect to the standard basis $\{1, i\}$, then finding the eigenvalues and eigenspaces of the matrix representation.

6. Solve the initial value problem

$$\begin{cases} y_1'' &= -2y_2 + \ \ y_1' + 2y_2' \\ y_2'' &= \ \ 2y_1 + 2y_1' - \ \ y_2' \\ & y_1(0) = 0, y_2(0) = 0, \\ & y_1'(0) = -3, y_2'(0) = 2. \end{cases}$$